

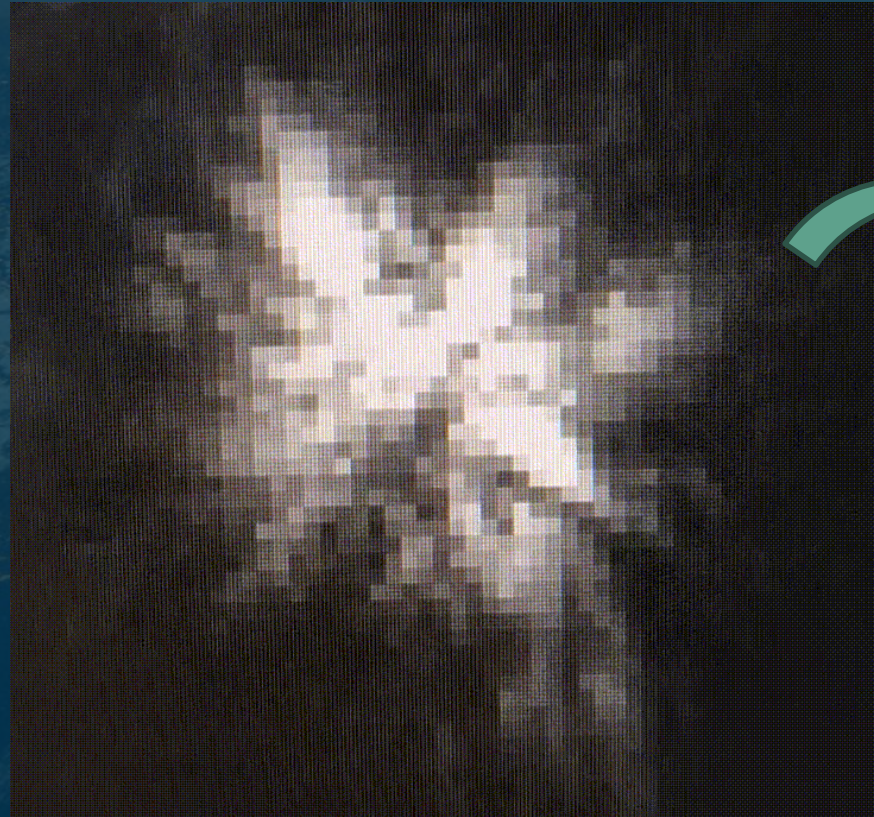
Introduction to Turbulence theory & Adaptive Optics

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Image of a star: Speckling & Dancing



Goal of Adaptive Optics is to compensate for atmospheric turbulence

1. General Introduction
2. Atmospheric turbulence
 - a) Introduction
 - b) Characterization of the turbulence
3. Adaptive Optics
 - a) Deformable mirrors
 - b) Wavefront Sensors
 - c) Control
 - d) Laser Guide Star
4. Examples

Introduction



Why do we need Adaptive Optics (AO) ?

Sometimes we don't, or we need a simple/partial AO compensation

→ The AO system depends on the application e.g:

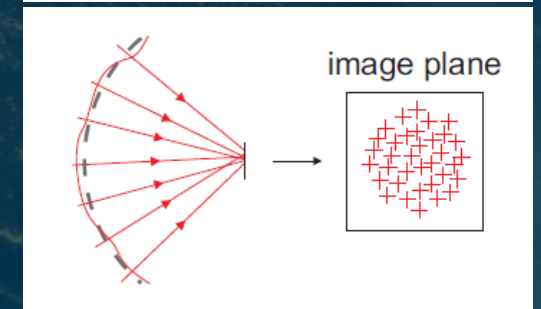
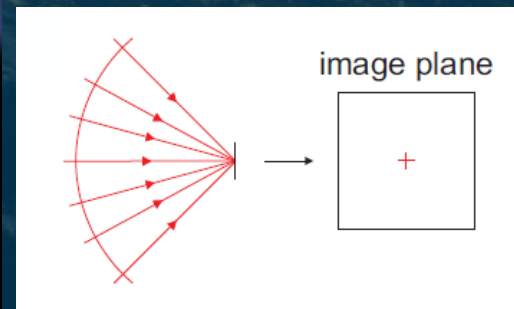
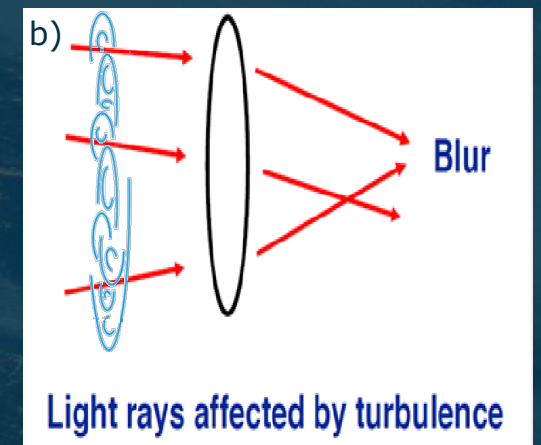
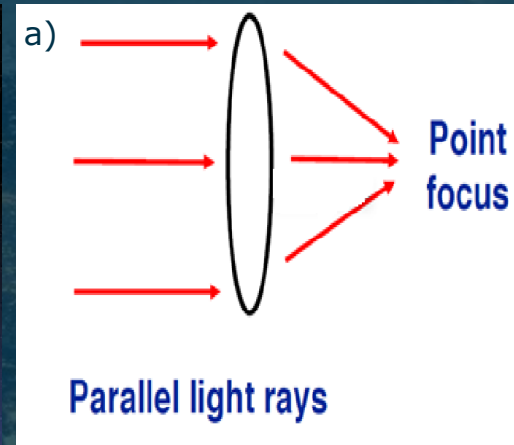
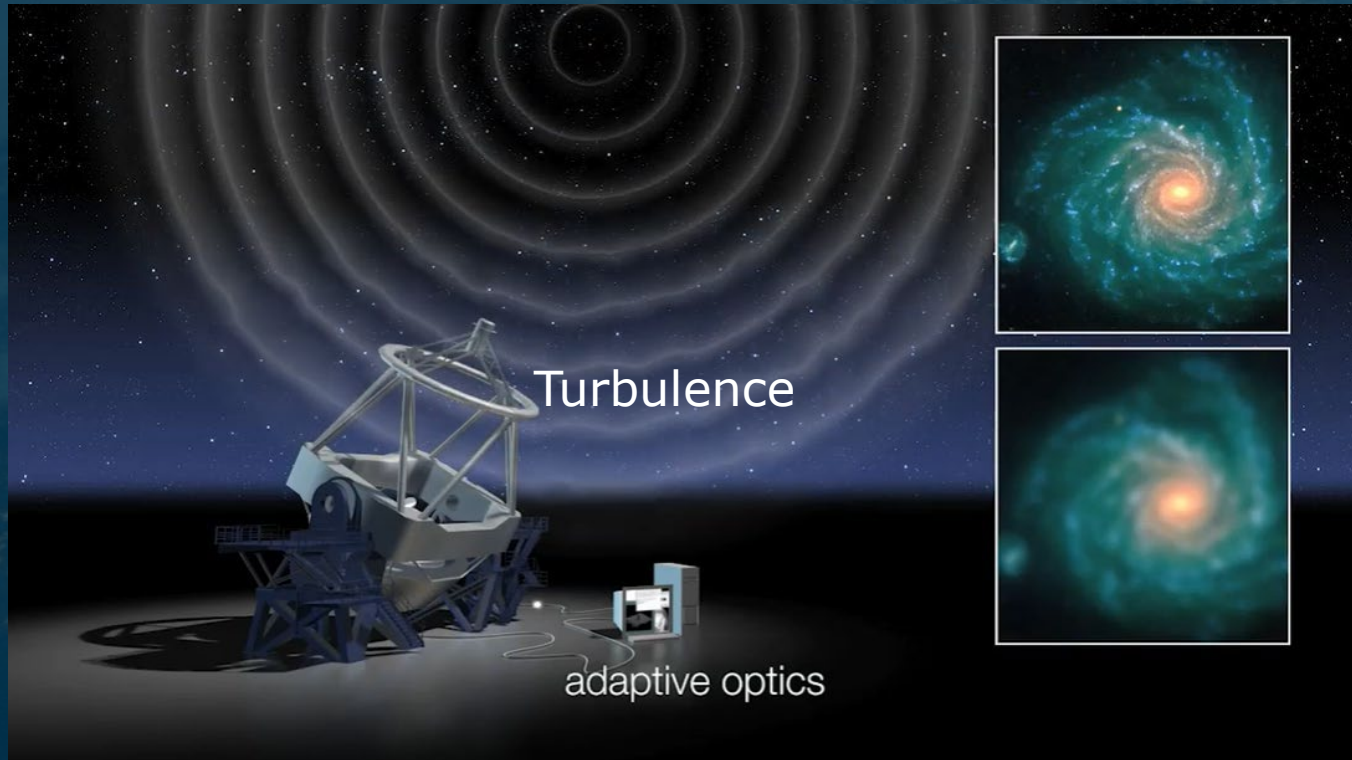
- Classical comm. / Quantum comm. ?
- Which data rates ?
- Which reliability ?
- Which environment (night, day, city center, rooftop etc) ?
- Which scenario (uplink, downlink, GEO, LEO etc) ?
- Which wavelength(s) ?
- Which telescope diameter ?
- Which budget ?
-

→ The AO system depends on the application

Impact of atmospheric turbulence (I)

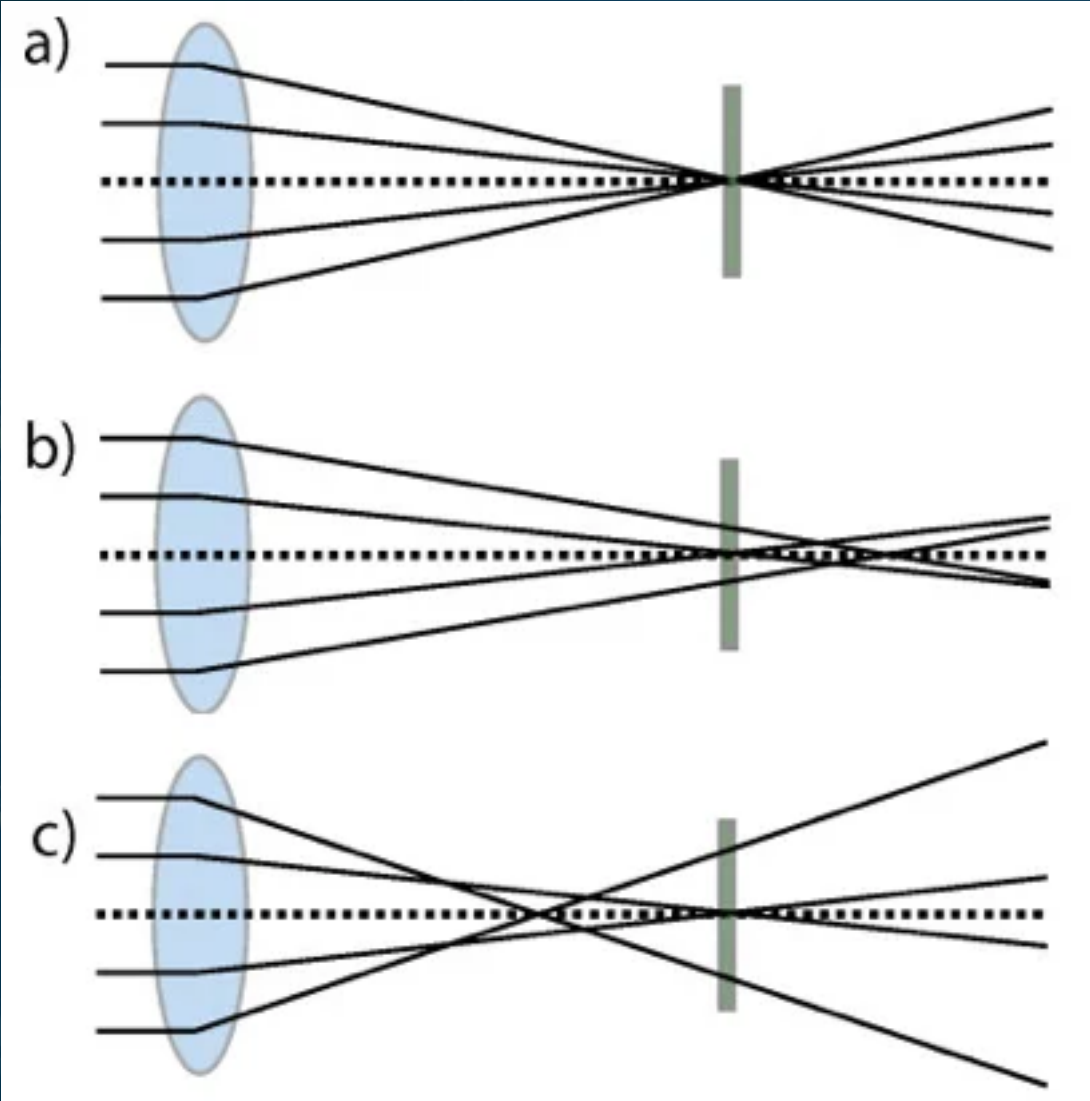
Light coming from a satellite get distorted because of atmospheric turbulence → Aberrations

→ The rays reaching the telescope are not anymore parallel → Cannot be focused in a sharp spot → Blurred spot



→ Compensate for the turbulence with Adaptive Optics

Aberrations (II)



Impact of atmospheric turbulence (II): up & down effects

Downlink:

- Phase distortion
- Spot "dancing"
- Speckling (random intensity)
- Deteriorates the coupling of the light in the optical fiber ($\approx 6\mu\text{m}$!) / detector

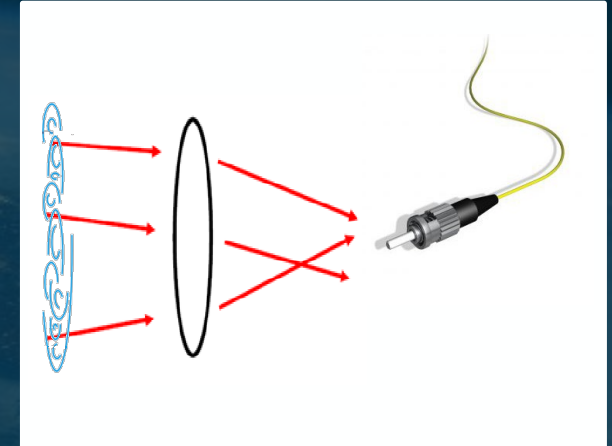
Uplink:

- Beam Wander (random deviation of the beam, can even miss the satellite !)
- Spreading of the beam
- Speckling (random intensity)
- Deteriorates the stability of the power received by the satellite

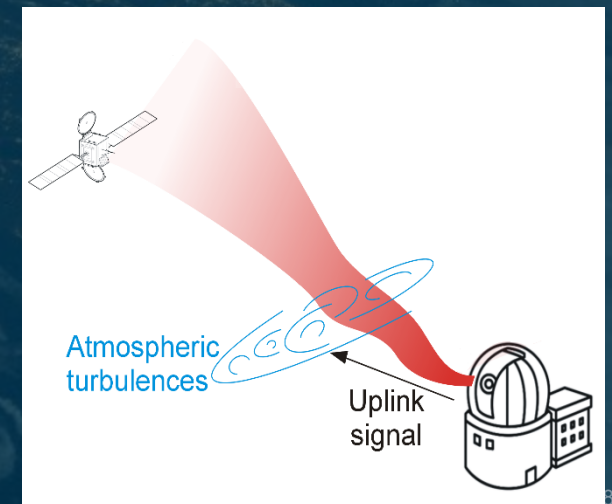
Impact: Fades/Surges in the signal → transmission errors !

→ Asymmetric effects for up & down links

Downlink

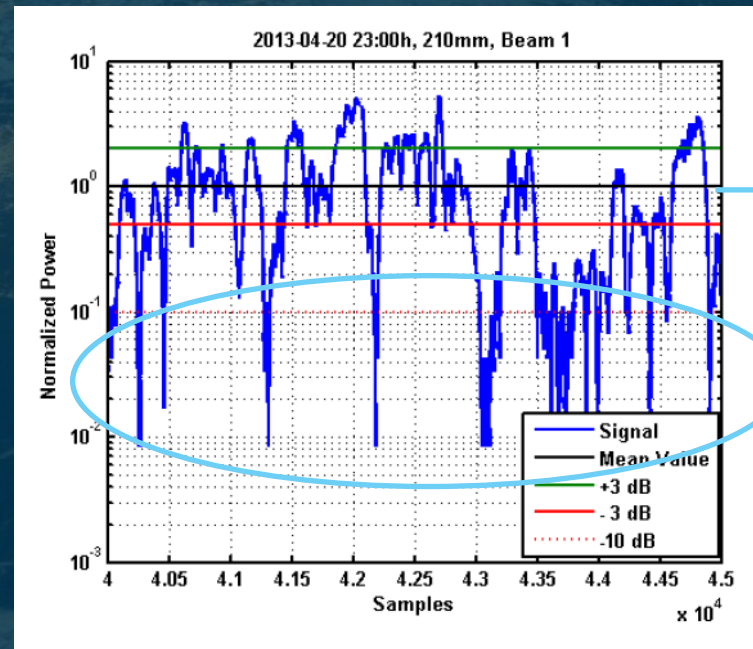
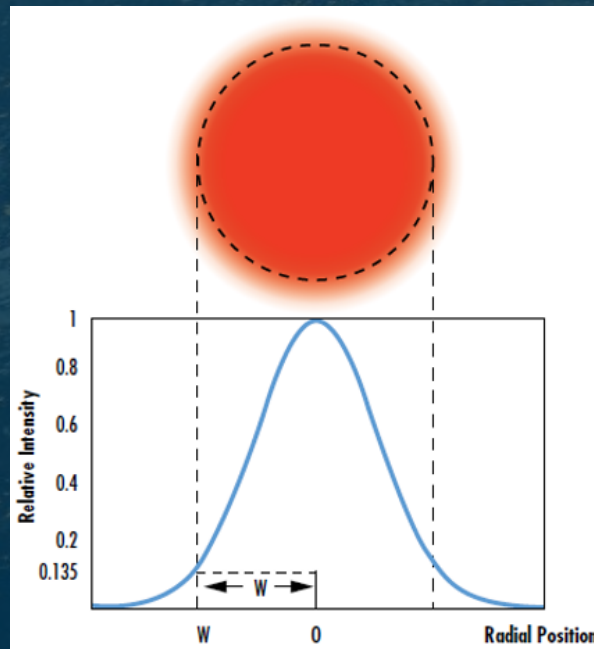


Uplink



Ideal Gaussian beam

Uplink power @ARTEMIS GEO

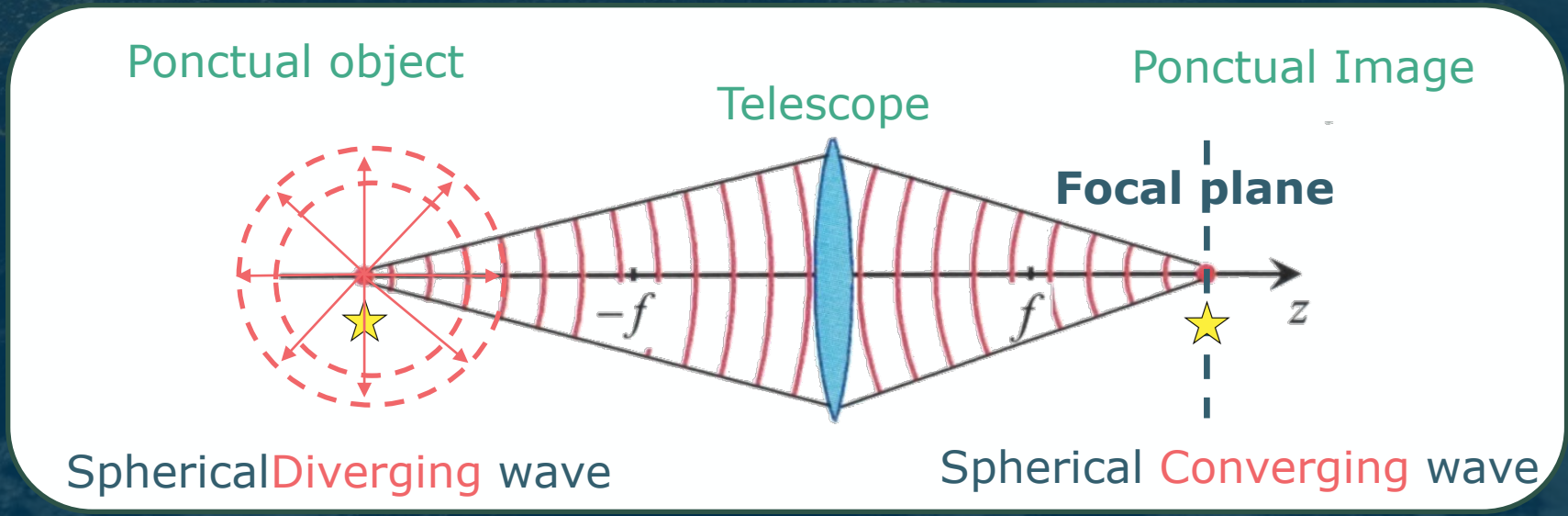


Average power

Fades

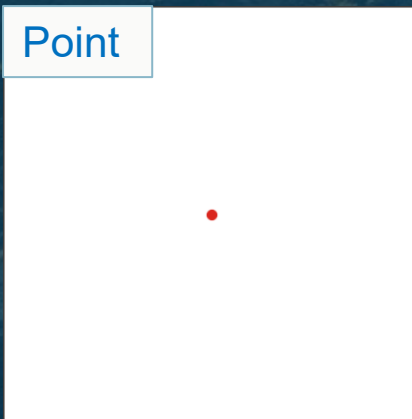
The average irradiance is important as well as its variance

- A telescope collect only a tiny part of the emitted light of source and focus it
- The bigger the telescope:
 - The more light (photons) it collects
 - The better is its resolution



Because a telescope cannot collect all the light emitted by a ponctual source, **the image on the detector is not a perfect point but an "Airy spot" → decreases the resolution**

Theoretical image



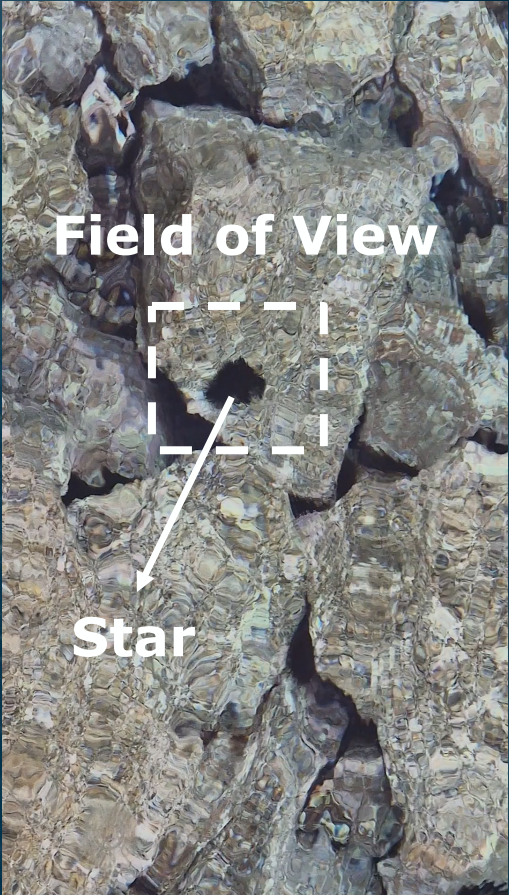
Actual image



Even without any turbulence, one cannot have a smaller spot than the Airy pattern.
→ This is the best one can achieve with AO

Atmospheric Turbulence

Turbulence



Chaotic movement of air, resulting in fluctuations of the atmospheric index of refraction

→ Wavefront error (phase) → Intensity fluctuations (e.g twinkling of the stars)

= Optical turbulence

- 2 required ingredients for turbulence:

- ΔT
- Wind

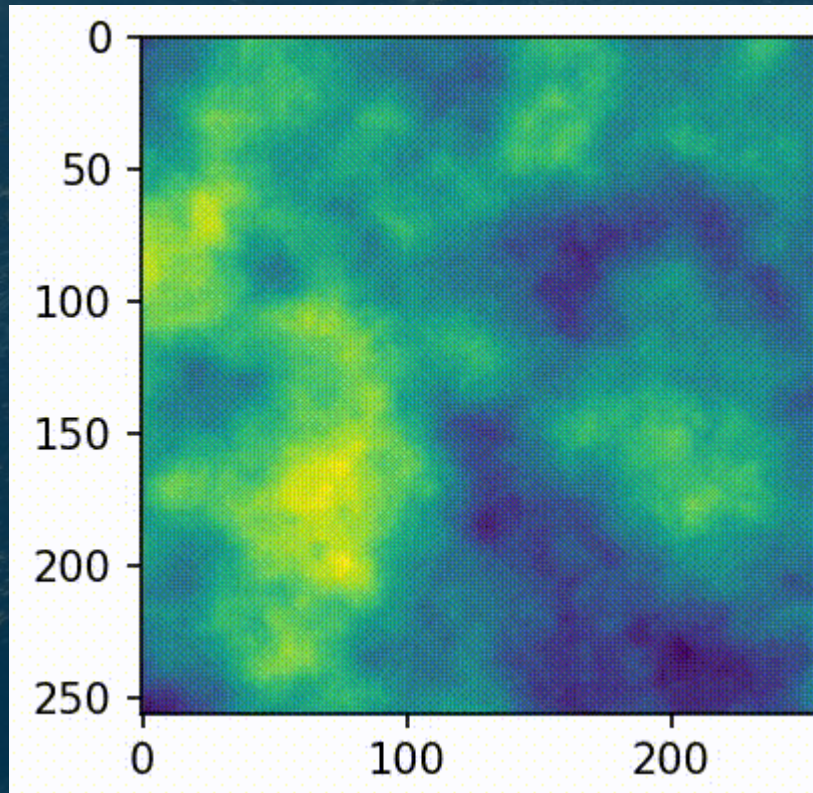
- 2 effects:

- Wavefront distortions (phase)
- Scintillation (Random optical lenses) (amplitude)

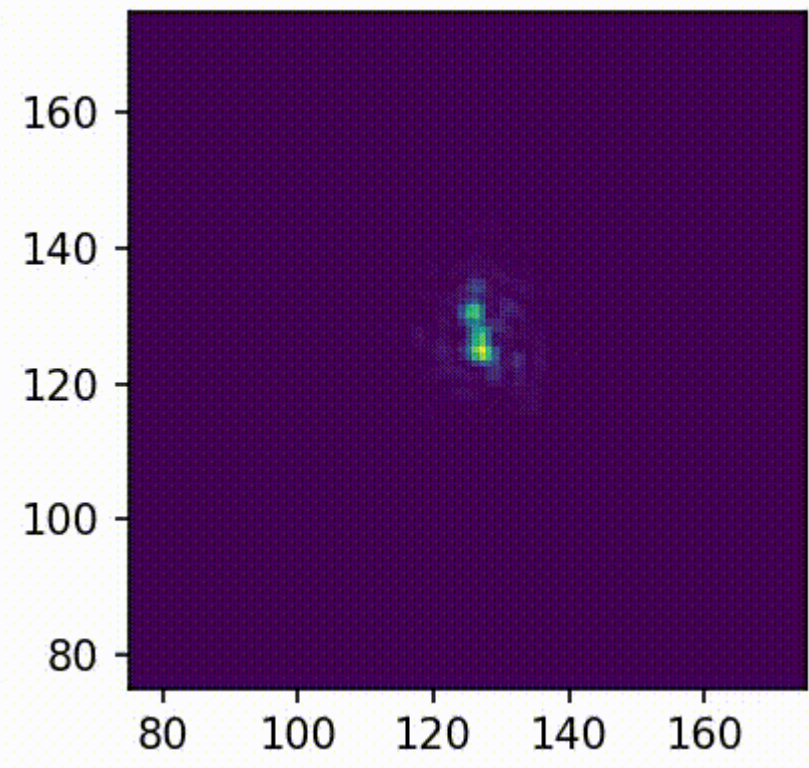


Atmospheric turbulence impacts both the phase & intensity of the beam

Phase error screen



Intensity profile after propagation through phase error screen



Kolmogorov theory of turbulence

Mathematical difficulty of atmospheric turbulence

Kolmogorov developed a statistical theory/description of turbulence (1941)

Atmosphere is a viscous fluid

Wind velocity  until Reynolds number exceeded

→ Creates **local unstable air masses** ("**Eddies**")

Under the influence of inertial forces, **Eddies break up into smaller eddies to form a continuum of eddy size** for the transfer of energy from a macroscale L_0 to a microscale l_0 , dissipated as heat due to friction.



Born	Andrey Nikolaevich Kolmogorov 25 April 1903 Tambov, Russian Empire
Died	20 October 1987 (aged 84) Moscow, Soviet Union
Citizenship	Soviet Union
Nationality	Soviet Union
Fields	Mathematics

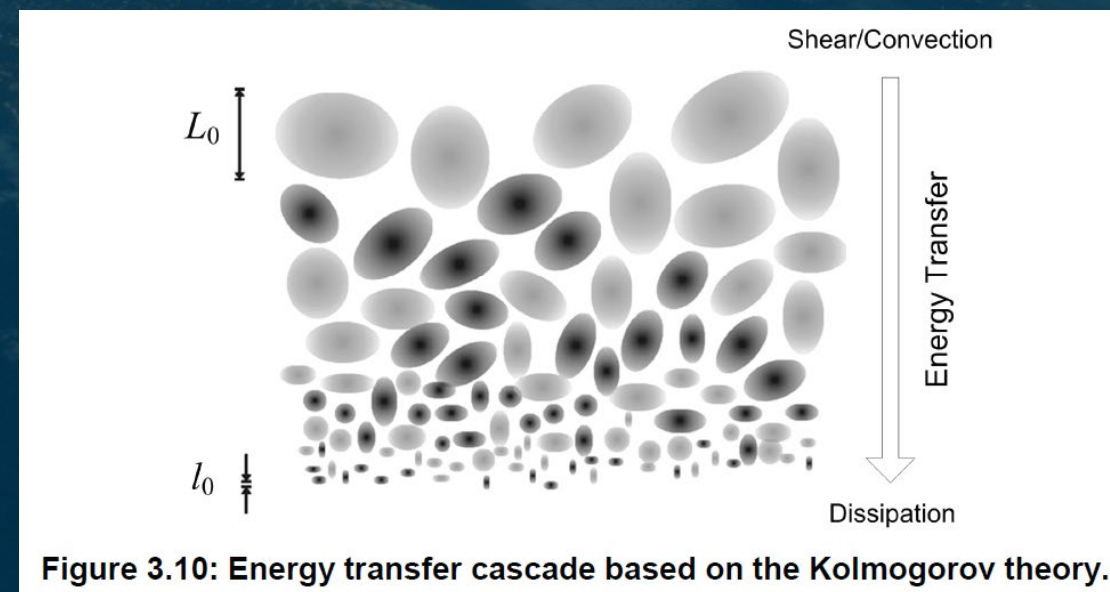
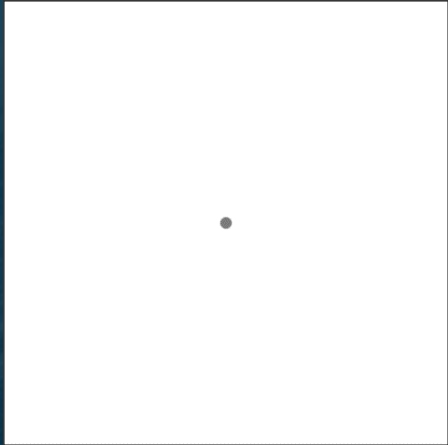


Figure 3.10: Energy transfer cascade based on the Kolmogorov theory.

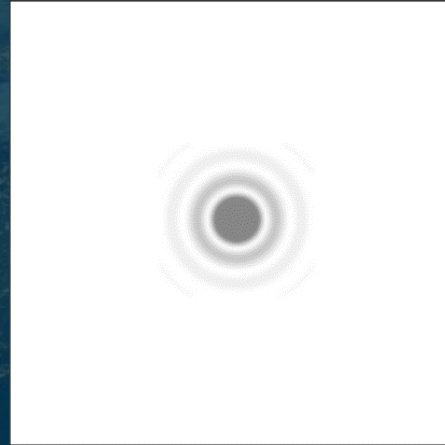
"Infinite" telescope

Perfect point



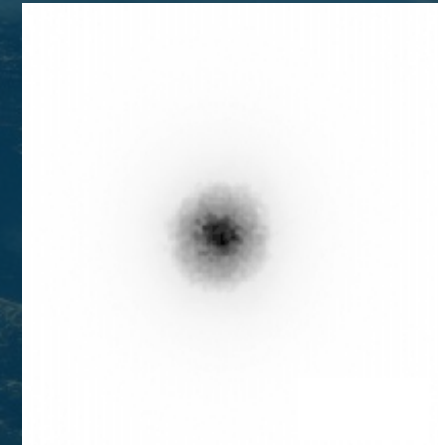
Real telescope **without** turbulence

Diffraction limited (Airy spot)



Real telescope **with** turbulence

Blurred spot



The seeing gives the strength of the turbulence

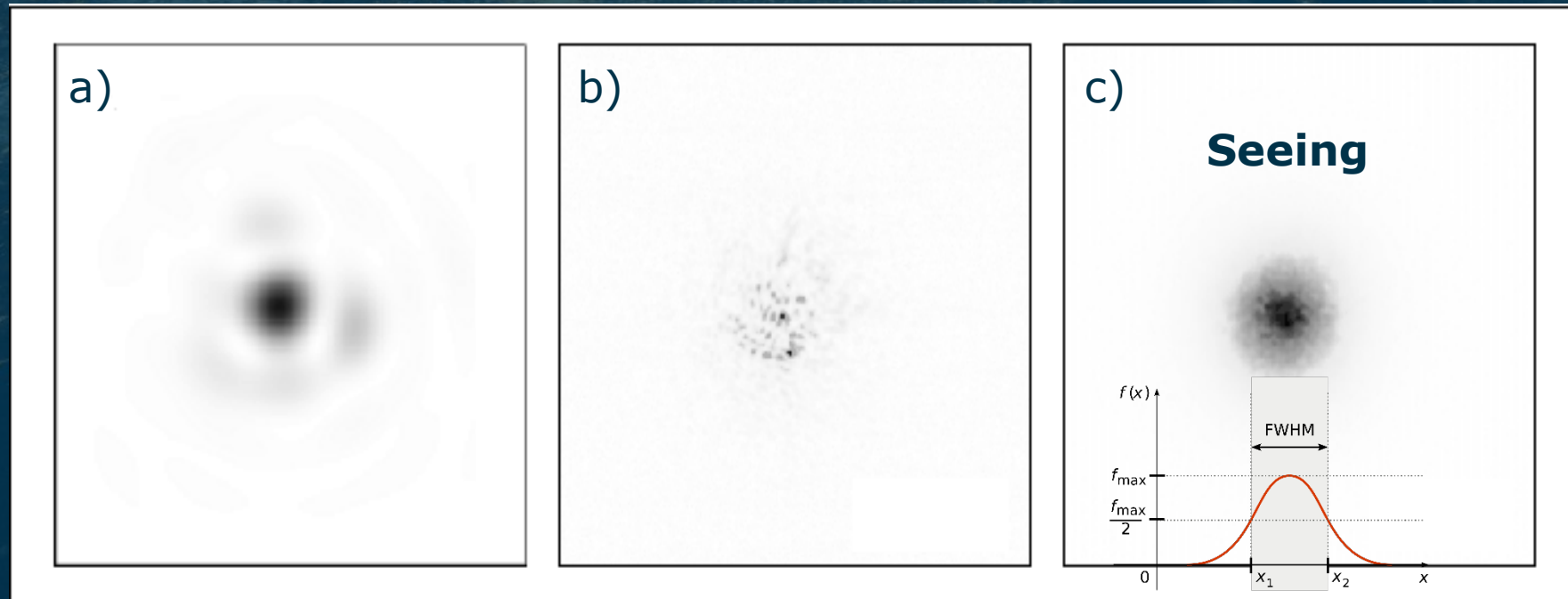
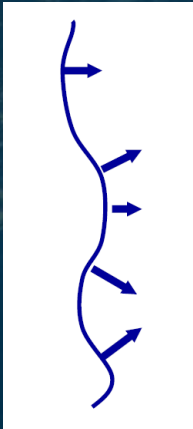
Seeing (ϵ_0) = FWHM of the long exposure spot

The seeing = achievable resolution of the telescope in presence of turbulence

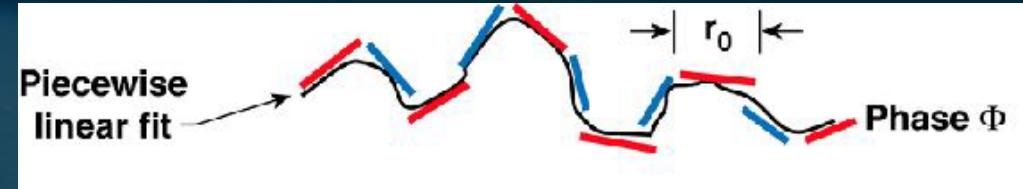
With turbulence: image of a star looks very different through telescopes of different apertures

- Small telescope: image looks like diffraction limited (figure a)) & "dance"
- Large telescope:
 - Short exposure: many corrugations but image frozen (figure b)) = speckles
 - Long exposure (seeing): many corrugations exposed over a long time (~ 1 min) produces a blurred image. Figure c).

Distorted wavefront



- r_0 = length of coherent cells (i.e rms WFE = 1 radian)
= integrated turbulence along the line of sight.

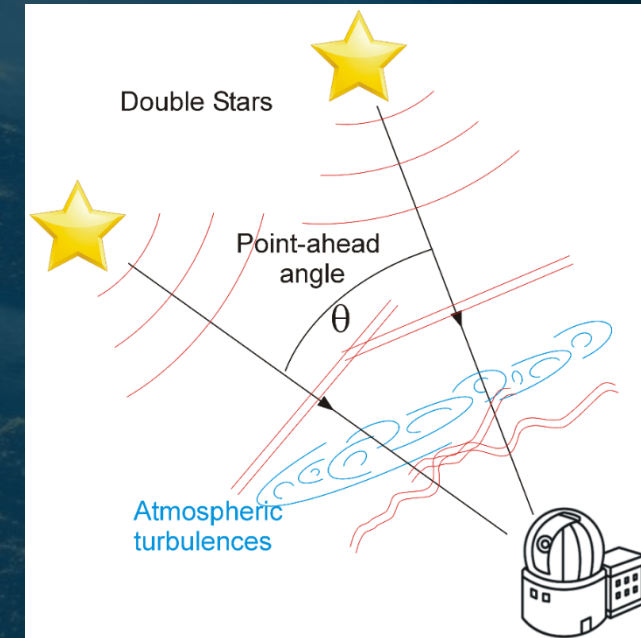


- Directly related to the seeing by: $r_0 = 0.98 \frac{\lambda}{\epsilon_0}$
- Small $r_0 \rightarrow$ Strong turbulence
- $r_0(\lambda_1) = r_0(\lambda_0) \left(\frac{\lambda_1}{\lambda_0}\right)^{6/5}$
- Typical values (at $\lambda = 500\text{nm}$) are $r_0 = 10\text{cm}$

r_0 is the length of a “flat” part of wavefront,
AO easier at large wavelengths

Without AO, a large telescope does not have a better resolution than a telescope of diameter r_0

- The wavefront error depends on the line of sight
- θ_0 is the angular separation in the sky at which 2 WFE are considered as nearly identical (1rad RMS)
- Small isoplanatic angle \rightarrow Strong turbulence
- $\theta_0(\lambda_1) = \theta_0(\lambda_0) \left(\frac{\lambda_1}{\lambda_0}\right)^{6/5}$
- Typical values (at $\lambda = 500\text{nm}$): a few arcsec



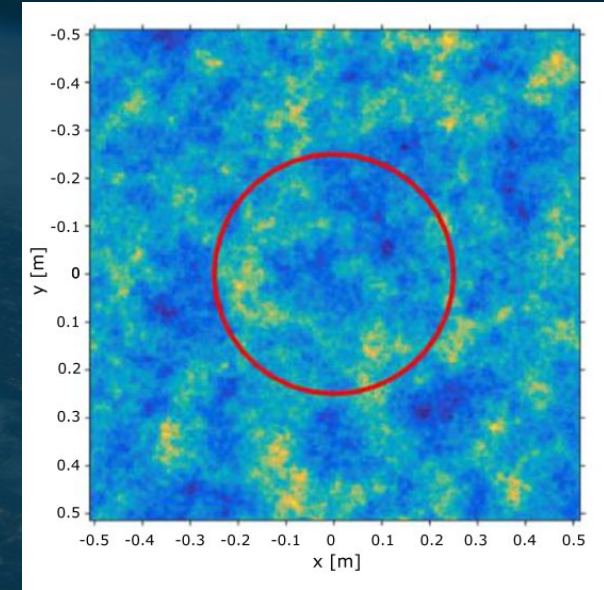
The isoplanatic angle is especially important for uplinks

- A measure of the timescale on which the wavefronts change by 1 rad RMS.
- The smaller the coherence time, the faster is the evolution of the turbulence, the more difficult it is to compensate for it.
- The larger the wavelength of interest, the larger τ_0 as: $\tau_0(\lambda_1) = \tau_0(\lambda_0) \left(\frac{\lambda_1}{\lambda_0}\right)^{6/5}$
- Typical values (at 500nm): a few milliseconds

The wavefront evolves in a very fast way : ~ ms !

Characterization of the turbulence (III): Scintillation index (σ_I)

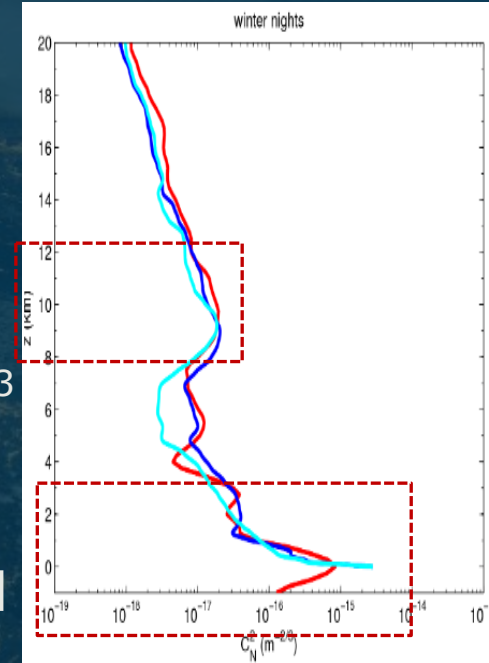
- The scintillation index is related to fluctuations in the intensity at any position on the wavefront. It is defined as the ratio $\sigma_I = \frac{\text{Var } I}{\langle I \rangle^2}$
- Large scintillation have impact on the optical fiber injection efficiency
- The bigger the telescope diameter the smaller the scintillation (aperture averaging !)
- $\sigma_I(\lambda_1) = \sigma_I(\lambda_0) \left(\frac{\lambda_1}{\lambda_0}\right)^{-7/6}$
- Typical values (at 500nm) are of the order of 10-20%.



The scintillation is less important for large telescopes because of spatial averaging

Characterization of the turbulence (III): Refractive index structure constant $C_n^2(h)$

- Measure of the strength of the optical turbulence as a function of the altitude
- The variance of the difference between two values of the refractive index is given by $D_N(\rho) = \langle |n(r) - n(r + \rho)|^2 \rangle = C_n^2 \rho^{2/3}$
- The integrated parameters can be calculated with $C_n^2(z)$
- Typical values of C_n^2 range from $10^{-13} \text{ m}^{-2/3}$ near the ground to $10^{-17} \text{ m}^{-2/3}$ an altitude of 10 km (for classical astronomical sites).
- For non-astronomical sites (e.g optical comm.), it can be much worse and depend much on the location & time



Most of the turbulence occurs at low altitudes (~ first kms)

$$r_0 = \left[0.423k^2 \sec(\gamma) \int_0^\infty C_n^2(h) dh \right]^{-3/5}$$

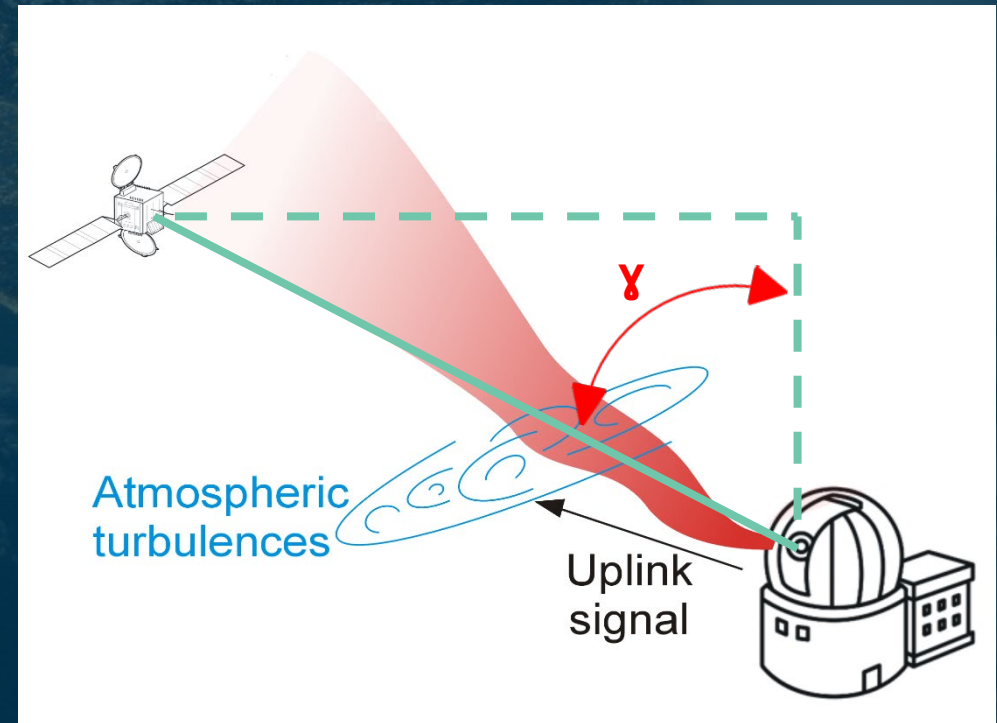
$$\theta_0 = \left[2.91k^2 \cos^{-8/3}(\gamma) \int_0^\infty C_n^2(h) h^{5/3} dh \right]^{-3/5}$$

$$\tau_0 = 0.314 \frac{r_0}{\bar{V}_{5/3}}$$

$$\bar{V}_{5/3} = \left[\frac{\int_0^\infty V(h)^{5/3} C_n^2(h) dh}{\int_0^\infty C_n^2(h) dh} \right]^{3/5}$$

$$\sigma_R^2 = 19.12\lambda^{-7/6} \sec^{11/6}(\gamma) \int_0^\infty h^{5/6} C_n^2(h) dh,$$

Dependency of the parameters as a function of the zenith angle γ (90°-elevation)



Equipment to monitor integrated & Cn2(h) parameters

Robotic equipment

First continuous measurements in urban environment during day & night!

Turbulence



Clouds



Aerosols & sky radiance



Deployed in Atlice-Sintra in June 2024

For 1 year campaign

GPS: 38°52'08.4"N 9°16'57.9"W

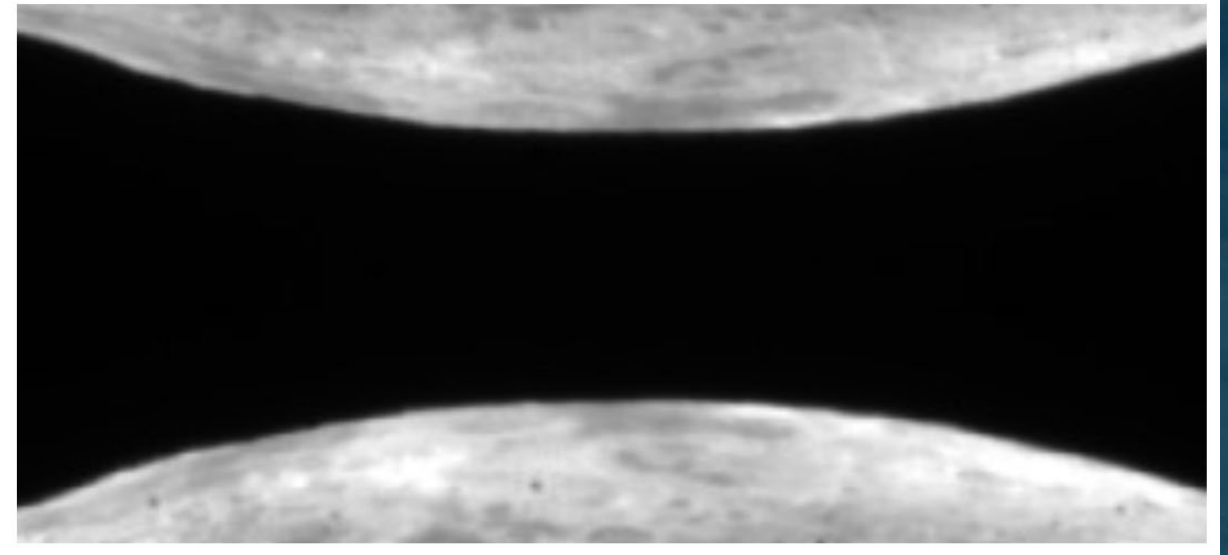
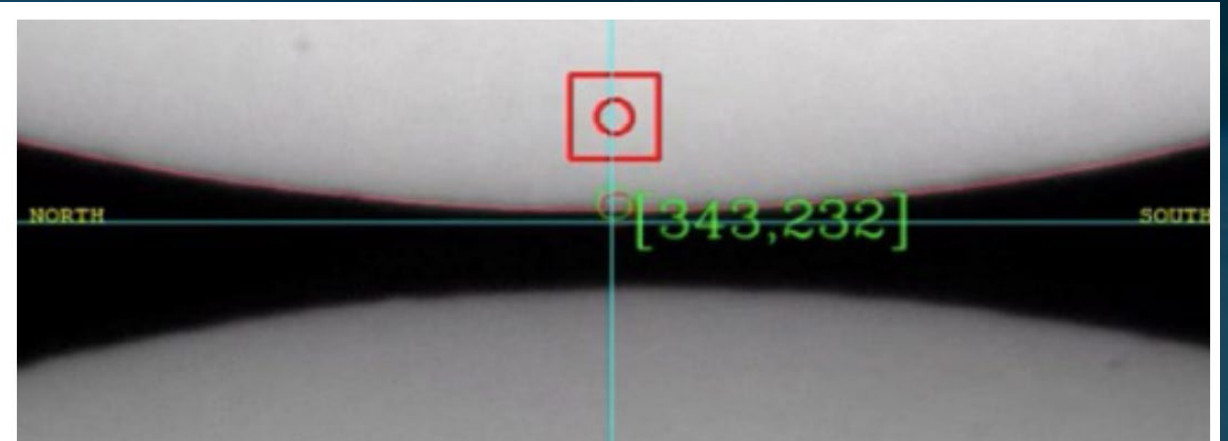
Similar equipment deployed in:

- Observatoire de la cote d'Azur, Fr
- Madrid, Spain
- Catania, Italy



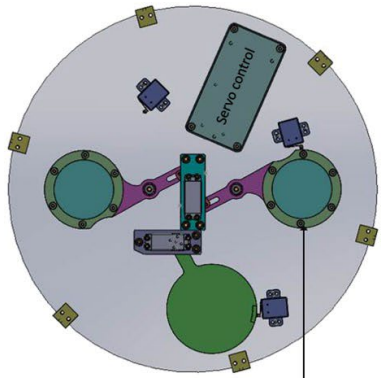
Several modes of operation:

- Nighttime: Moon limb
- Nighttime: Stars
- Daytime: Sun limb



Daytime PML

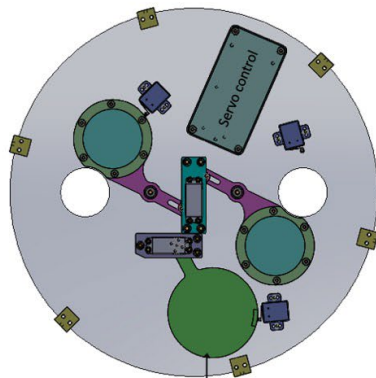
Target: Sun limb



Solar filter

Nighttime PML

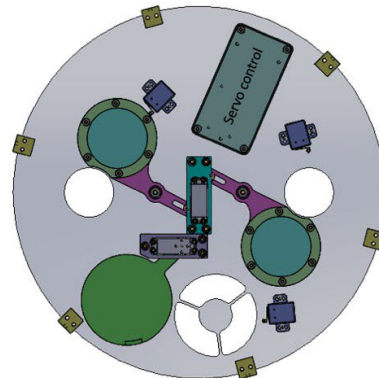
Target: Moon limb



Shutter

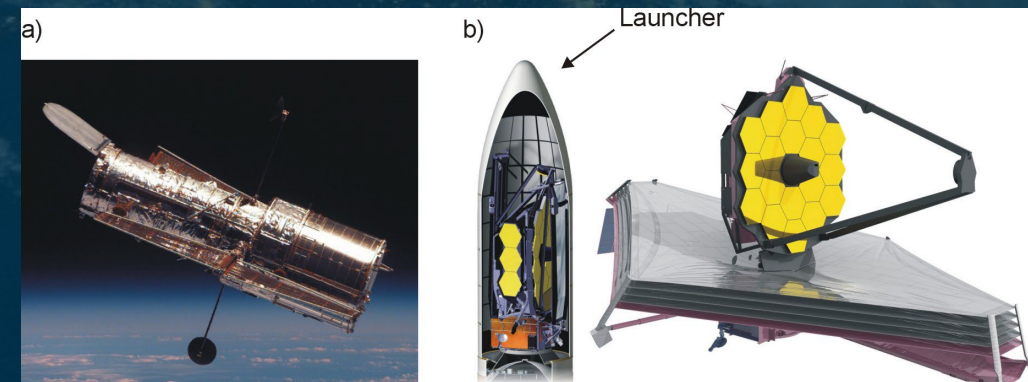
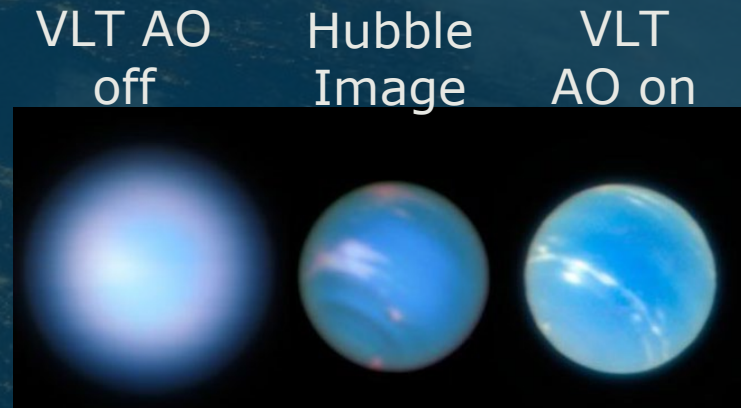
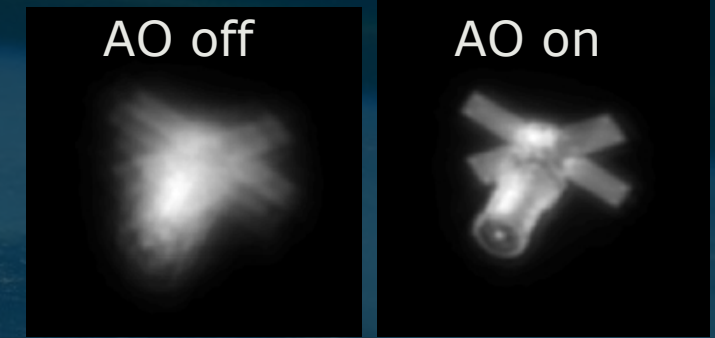
GDIMM: nighttime

Target: Bright stars



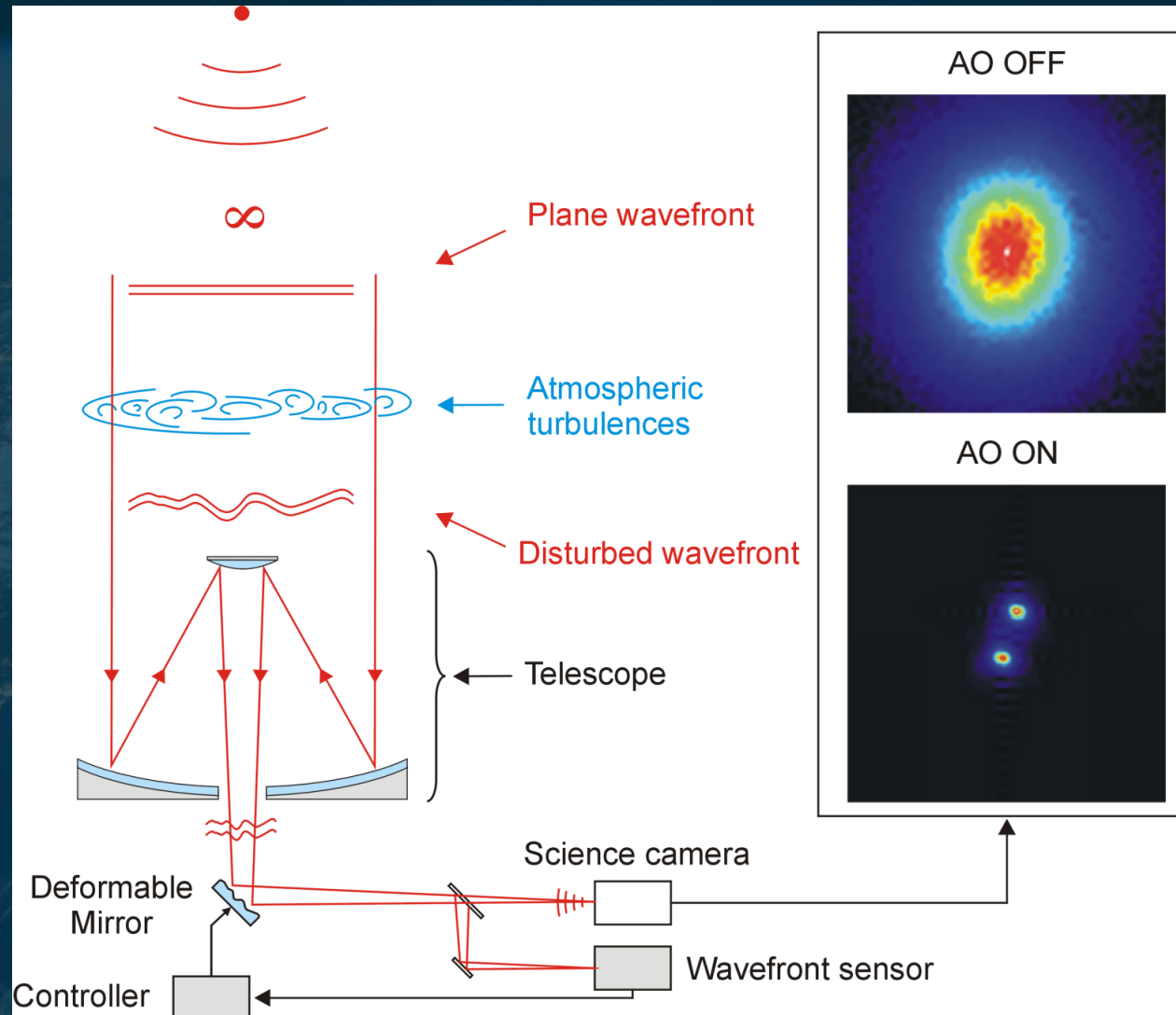
Adaptive Optics

- Atmospheric turbulence effects were already observed by Aristote (350BC) observing the twinkling of stars
- AO aims at compensating turbulence induced aberrations
- First envisioned by W.Babcock in 1953 for astronomy
- Initial developments for military for satellite tracking
- From 1990s developments for telescopes
- Many applications e.g: Astronomy, Optical communication, Ophthalmology, Microscopy
- AO can be considered for space to compensate for T° distortion, gravity release, manufacturing errors

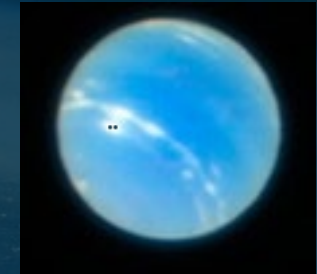


Key blocks of an AO loop:

1. Deformable Mirror
2. Wavefront sensor
3. Real Time Control
4. Reference in the sky



Initial image



Blurred image due to turbulence



Restored image Adaptive optics



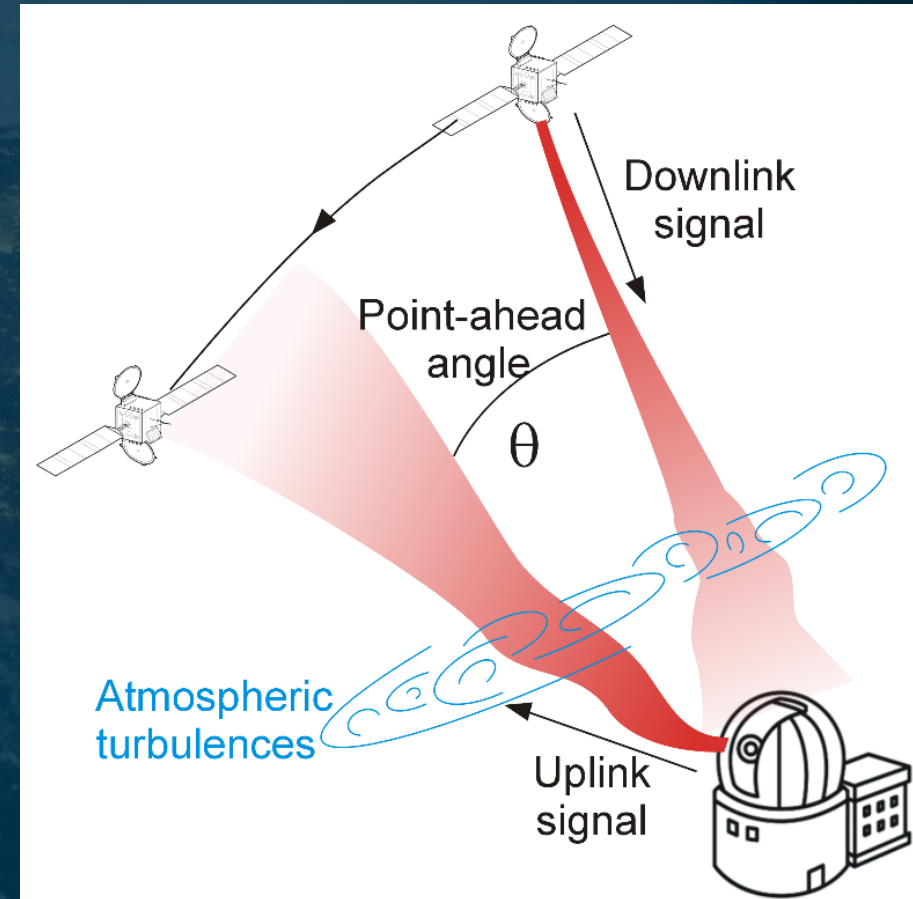
For downlinks:

One wants to (**post**) compensate the wavefront distortion

For uplinks:

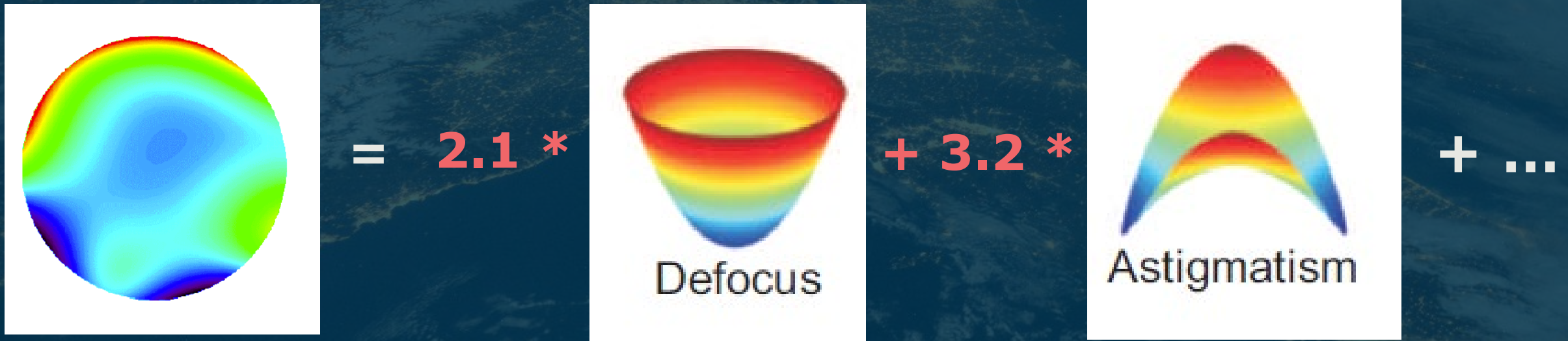
One wants to (**pré**) compensate the wavefront distortion

Issue: Point ahead angle

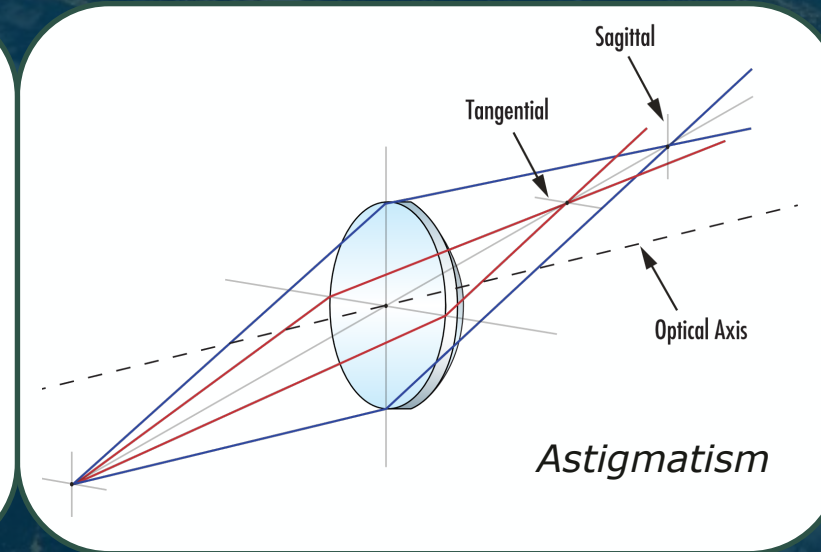
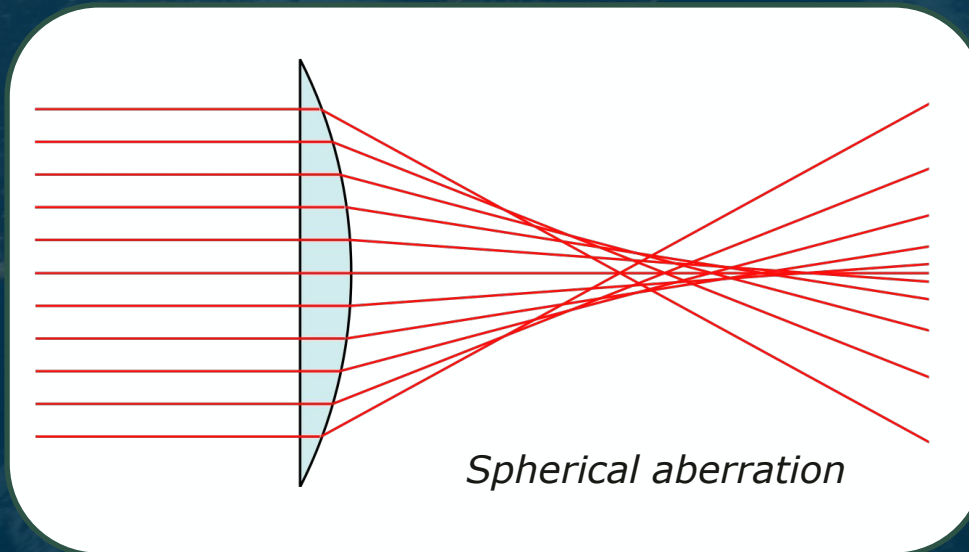
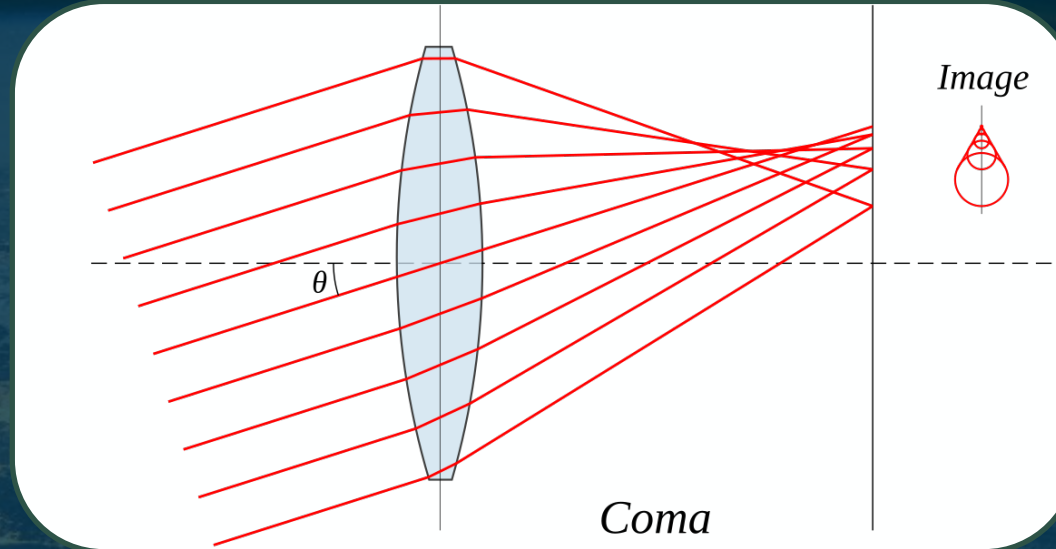
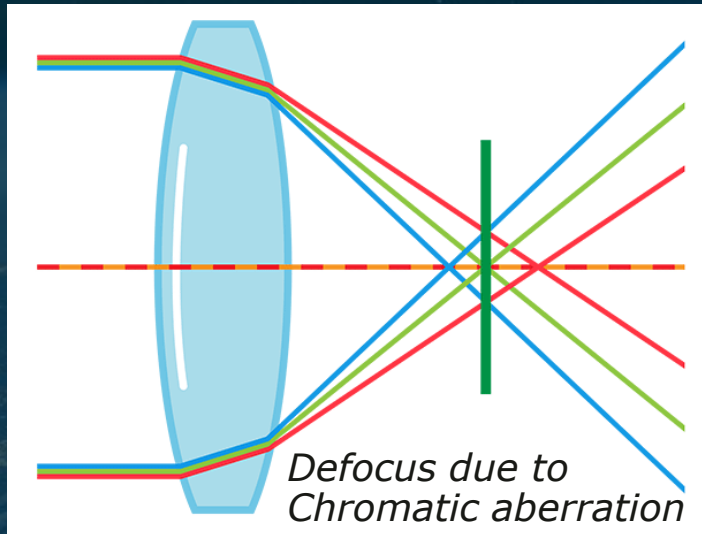


Any wavefront distortion can be decomposed in a set of orthogonal & complete basis e.g Zernike Modes

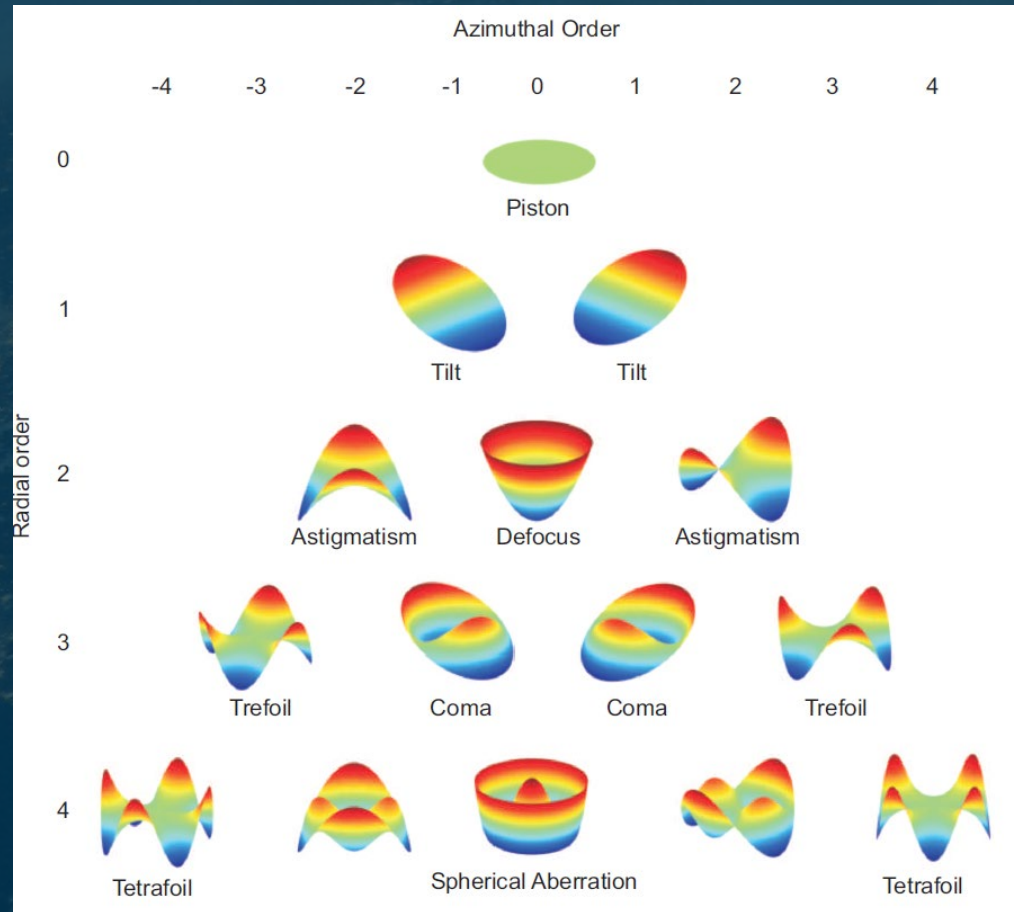
Random phase



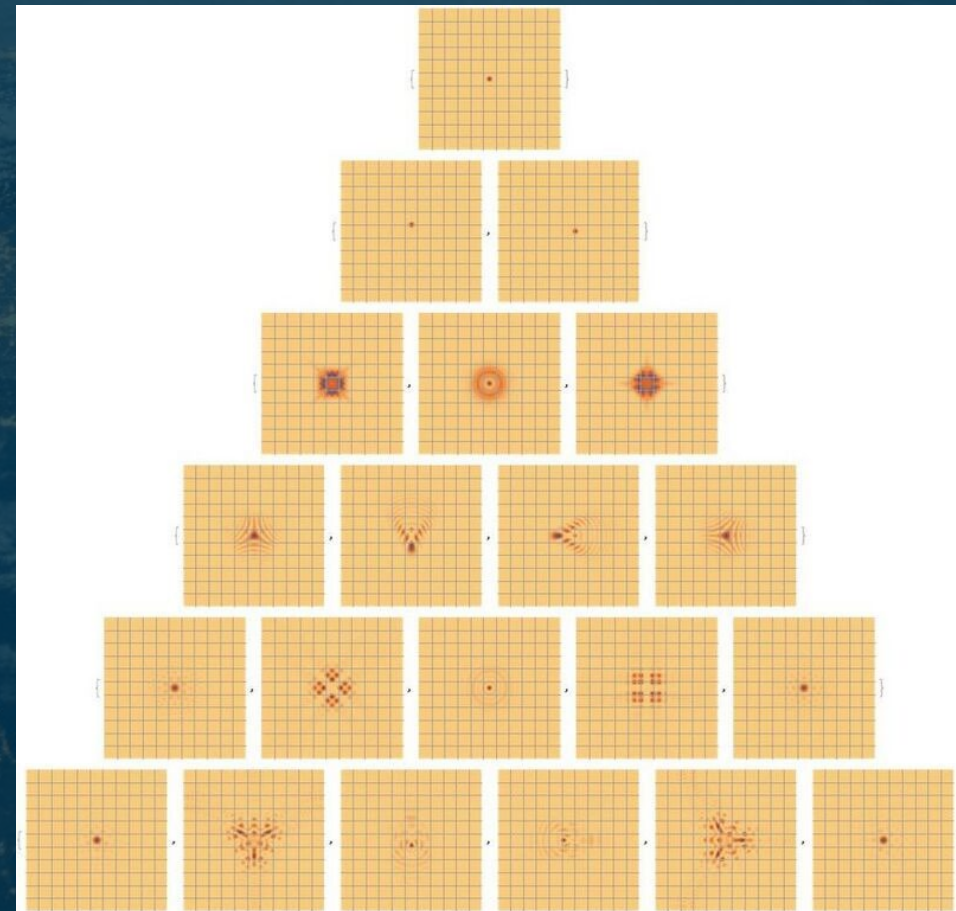
Zernike modes (II): Aberrations



At the entrance of the telescope

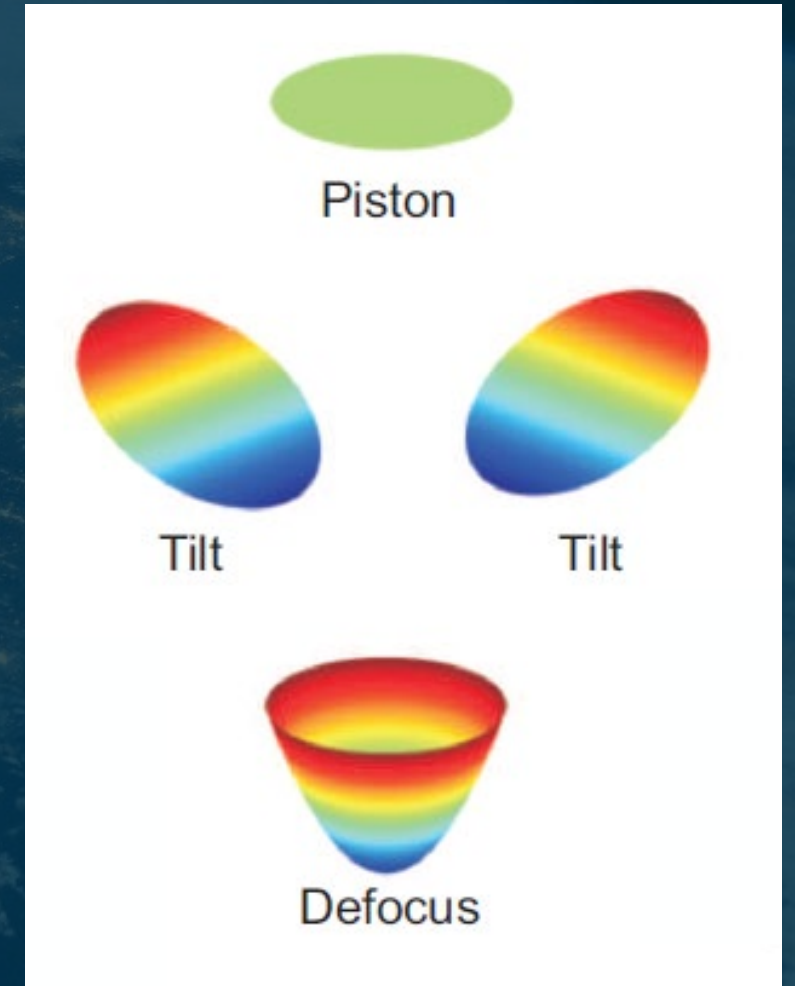


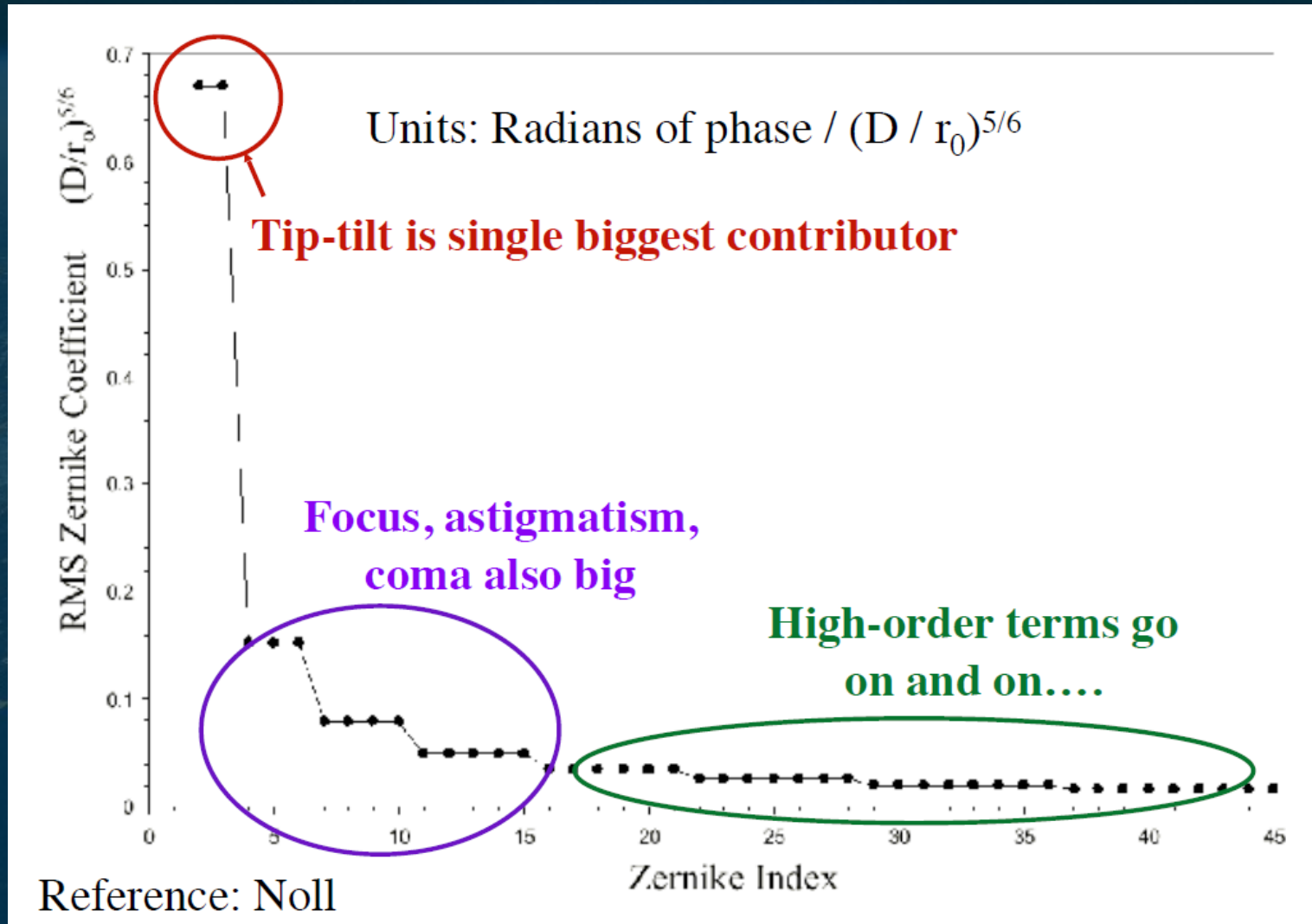
In the focal plane
(focused on the detector)



Important examples:

- Piston = Just a constant phase, often neglected
- Tilt = Angle of arrival of the wavefront
- Defocus = wrong focus (like in cameras)





$$Z_i(r, \theta) = \begin{cases} \sqrt{2(n+1)} R_n^m(r) G^m(\theta) & m \neq 0 \\ R_n^0(r) & m = 0 \end{cases}$$

$$R_n^m(r) = \sum_{s=0}^{(n-m)/2} \frac{(-1)^s (n-s)!}{s! \left(\frac{n+m}{2} - s\right)! \left(\frac{n-m}{2} - s\right)!} r^{n-2s}$$

$$G^m(\theta) = \begin{cases} \sin(m\theta) & i \text{ odd} \\ \cos(m\theta) & i \text{ even.} \end{cases}$$

Table 5.2 The first 36 Zernike polynomials

n	m	i	$Z_n^m(r, \theta)$	Name
0	0	1	1	piston
1	1	2	$2r \cos \theta$	x tilt
1	1	3	$2r \sin \theta$	y tilt
2	0	4	$\sqrt{3}(2r^2 - 1)$	defocus
2	2	5	$\sqrt{6}r^2 \sin(2\theta)$	y primary astigmatism
2	2	6	$\sqrt{6}r^2 \cos(2\theta)$	x primary astigmatism
3	1	7	$\sqrt{8}(3r^3 - 2r) \sin \theta$	y primary coma
3	1	8	$\sqrt{8}(3r^3 - 2r) \cos \theta$	x primary coma
3	3	9	$\sqrt{8}r^3 \sin(3\theta)$	y trefoil
3	3	10	$\sqrt{8}r^3 \cos(3\theta)$	x trefoil
4	0	11	$\sqrt{5}(6r^4 - 6r^2 + 1)$	primary spherical
4	2	12	$\sqrt{10}(4r^4 - 3r^2) \cos(2\theta)$	x secondary astigmatism
4	2	13	$\sqrt{10}(4r^4 - 3r^2) \sin(2\theta)$	y secondary astigmatism
4	4	14	$\sqrt{10}r^4 \cos(4\theta)$	x tetrafoil
4	4	15	$\sqrt{10}r^4 \sin(4\theta)$	y tetrafoil
5	1	16	$\sqrt{12}(10r^5 - 12r^3 + 3r) \cos \theta$	x secondary coma
5	1	17	$\sqrt{12}(10r^5 - 12r^3 + 3r) \sin \theta$	y secondary coma
5	3	18	$\sqrt{12}(5r^5 - 4r^3) \cos(3\theta)$	x secondary trefoil
5	3	19	$\sqrt{12}(5r^5 - 4r^3) \sin(3\theta)$	y secondary trefoil
5	5	20	$\sqrt{12}r^5 \cos(5\theta)$	x pentafoil
5	5	21	$\sqrt{12}r^5 \sin(5\theta)$	y pentafoil
6	0	22	$\sqrt{7}(20r^6 - 30r^4 + 12r^2 - 1)$	secondary spherical
6	2	23	$\sqrt{14}(15r^6 - 20r^4 + 6r^2) \sin(2\theta)$	y tertiary astigmatism
6	2	24	$\sqrt{14}(15r^6 - 20r^4 + 6r^2) \cos(2\theta)$	x tertiary astigmatism
6	4	25	$\sqrt{14}(6r^6 - 5r^4) \sin(4\theta)$	y secondary tetrafoil
6	4	26	$\sqrt{14}(6r^6 - 5r^4) \cos(4\theta)$	x secondary tetrafoil
6	6	27	$\sqrt{14}r^6 \sin(6\theta)$	
6	6	28	$\sqrt{14}r^6 \cos(6\theta)$	
7	1	29	$4(35r^7 - 60r^5 + 30r^3 - 4r) \sin \theta$	y tertiary coma
7	1	30	$4(35r^7 - 60r^5 + 30r^3 - 4r) \cos \theta$	x tertiary coma
7	3	31	$4(21r^7 - 30r^5 + 10r^3) \sin(3\theta)$	
7	3	32	$4(21r^7 - 30r^5 + 10r^3) \cos(3\theta)$	
7	5	33	$4(7r^7 - 6r^5) \sin(5\theta)$	
7	5	34	$4(7r^7 - 6r^5) \cos(5\theta)$	
7	7	35	$4r^7 \sin(7\theta)$	
7	7	36	$4r^7 \cos(7\theta)$	
8	0	37	$3(70r^8 - 140r^6 + 90r^4 - 20r^2 + 1)$	tertiary spherical

$$\sigma^2 = 1.03 \left(\frac{D}{r_0} \right)^{\frac{5}{3}}$$

ratio D/r_0 matters, Large D means large WFE

$$W(r, \theta) = \sum_{i=1}^{\infty} a_i Z_i(r, \theta)$$

WFE can be decomposed in Zernike modes
(complete & orthogonal polynomials over circular aperture)

Not the only possible base (e.g Karhunen-Loève)

$$\sigma^2 = \sum_{i=2}^{\infty} a_i^2$$

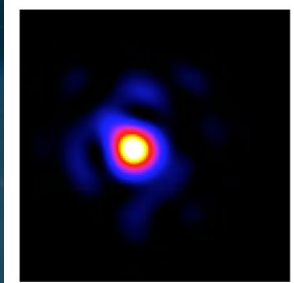
Property: WFE can be expressed as the sum of the square of the coeff.

$$\sigma_{N+1}^2 = \sum_{j=N+2}^{\infty} c_{jj} (D/r_0)^{5/3}$$

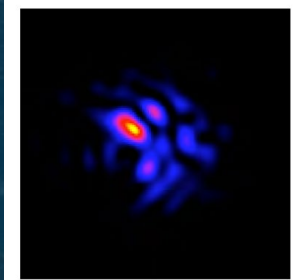
Residual WFE after N modes compensated. C_{jj} tabulated (Noll 1976)

$$\sigma_3^2 = 0.134 (D/r_0)^{5/3}$$

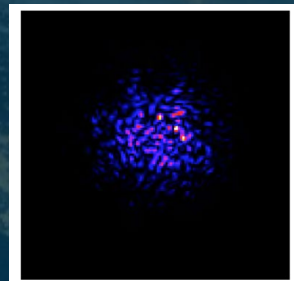
After removing the Piston, Tip and Tilt modes (biggest contribution !)



D = 1 m

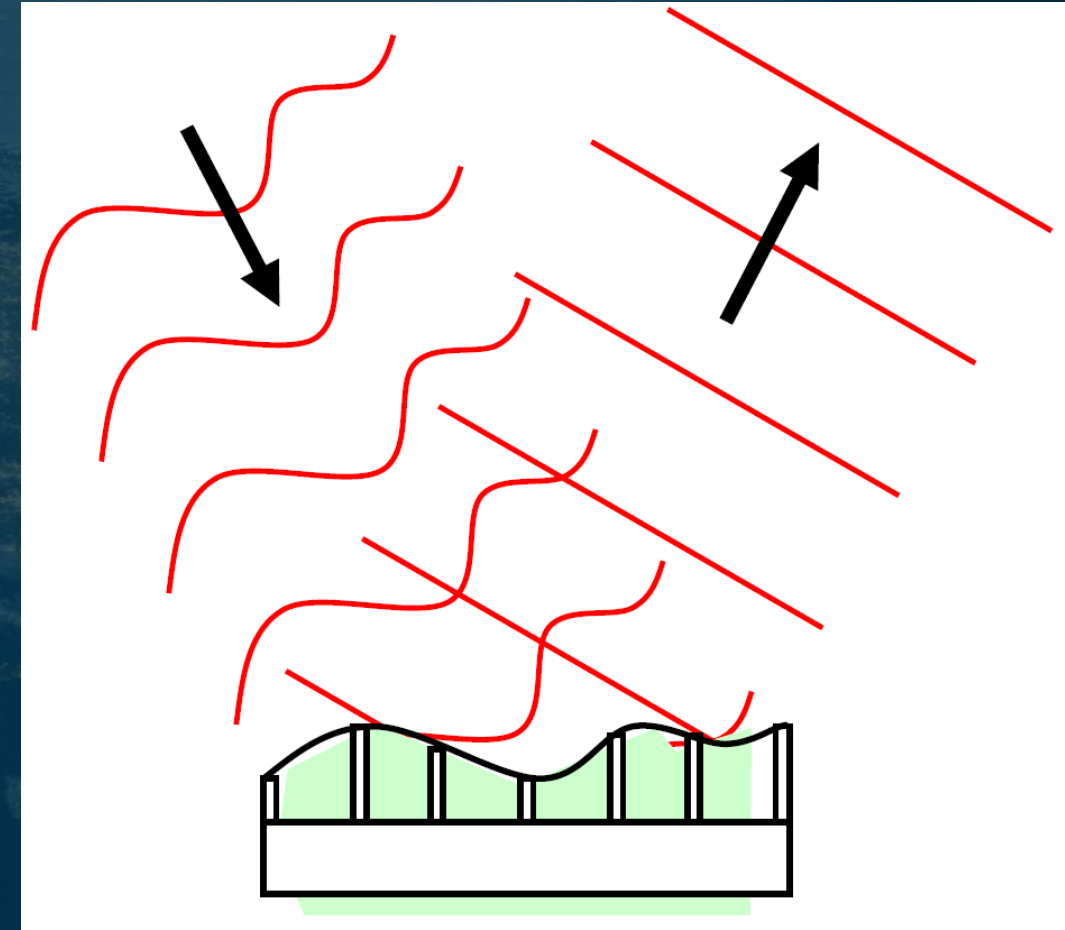
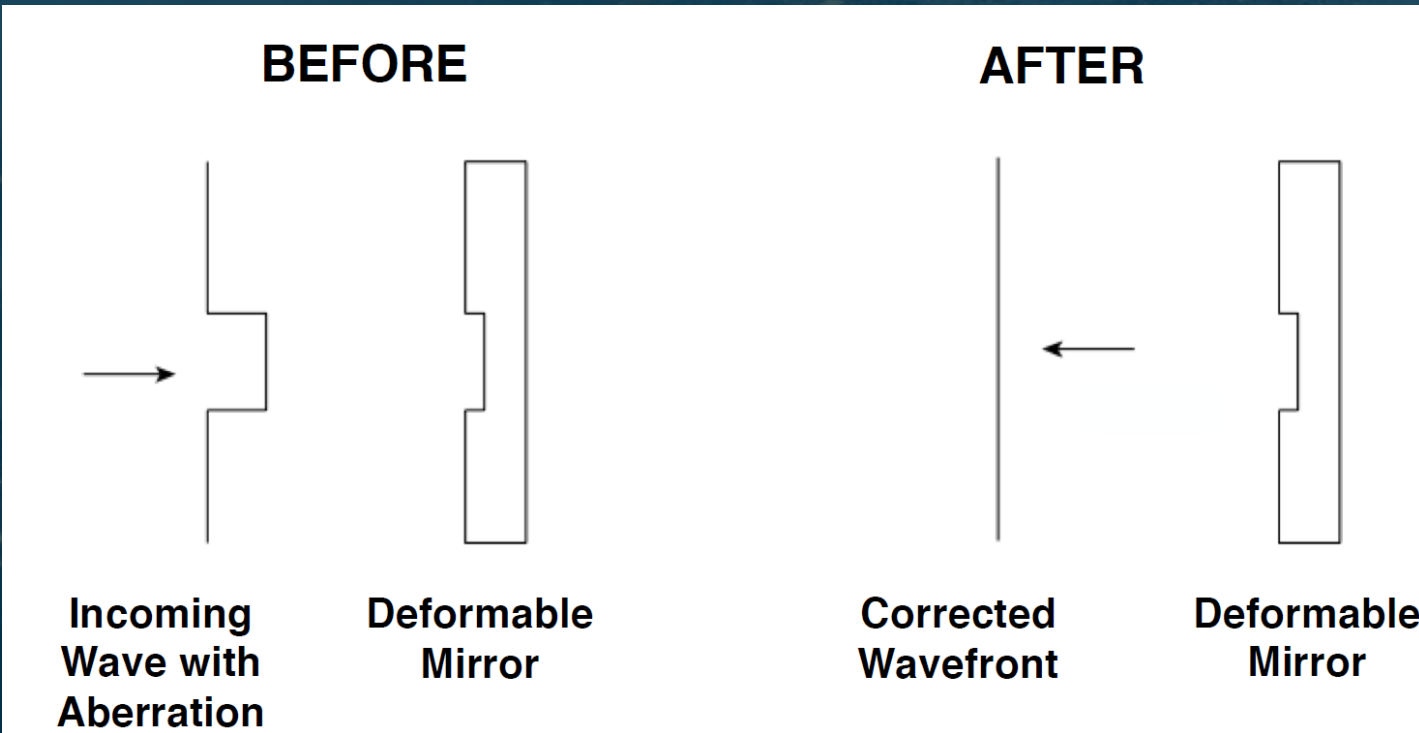


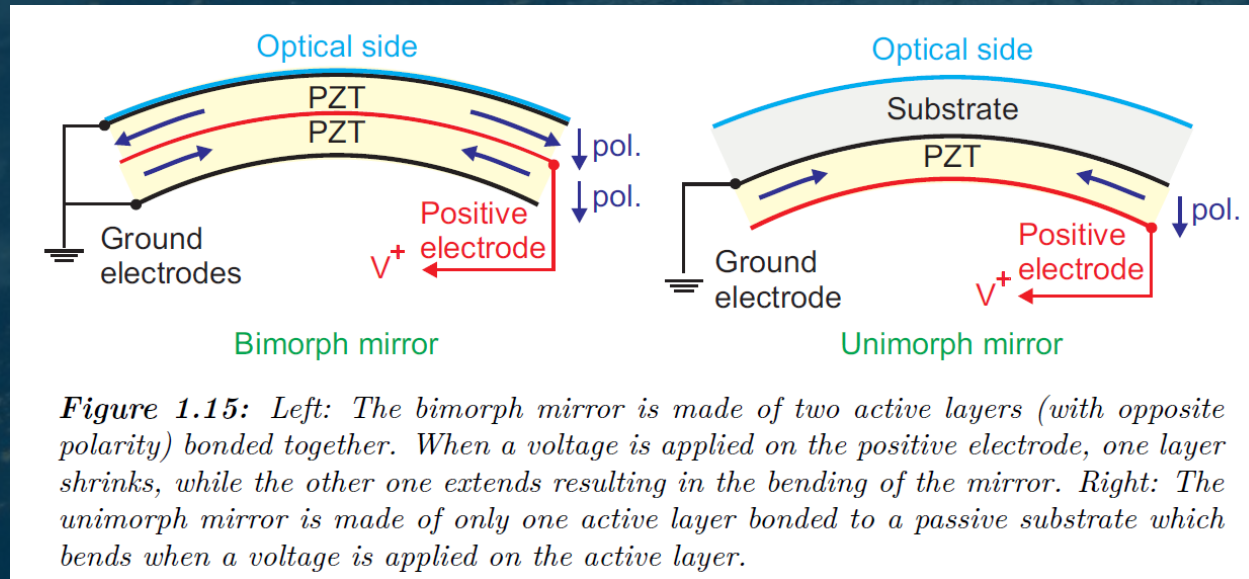
D = 2 m



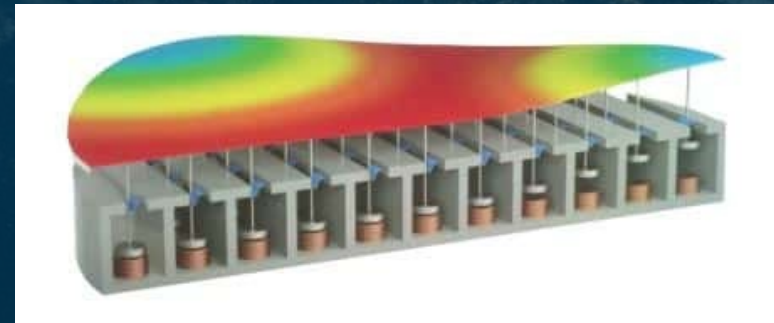
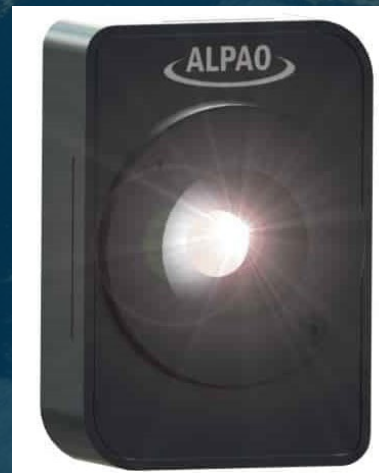
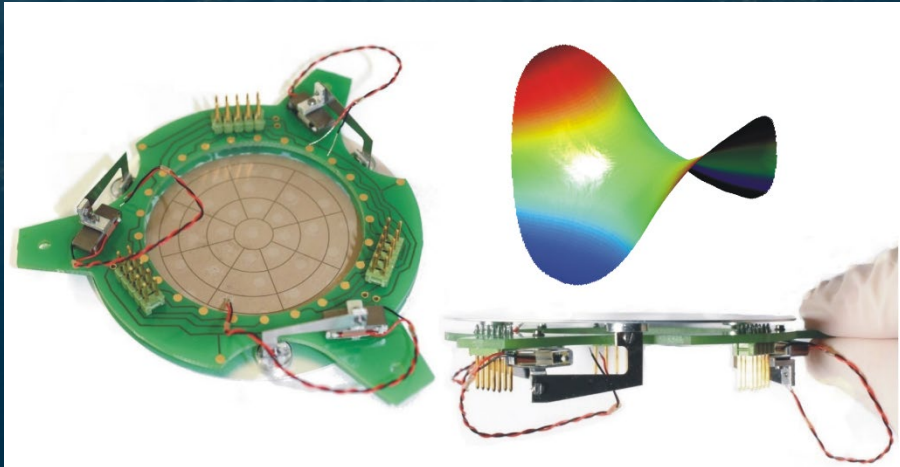
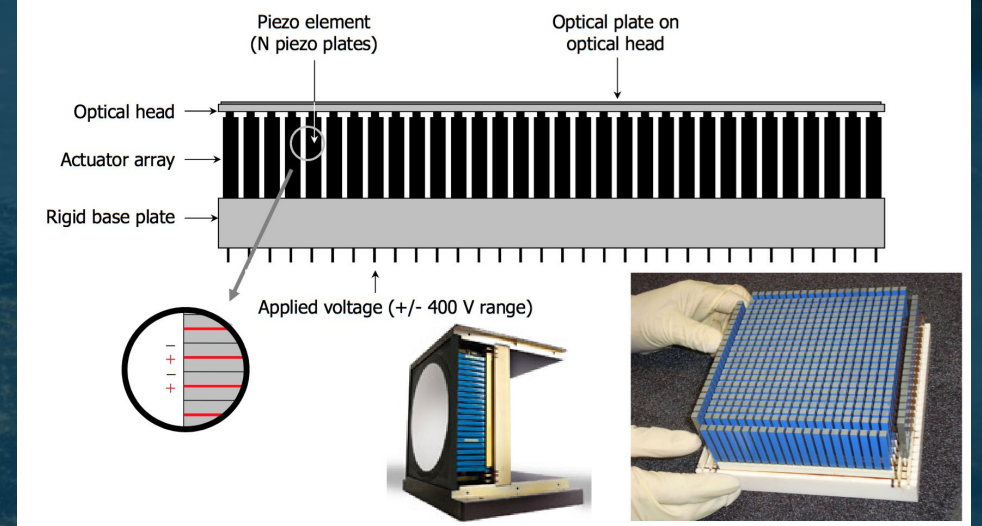
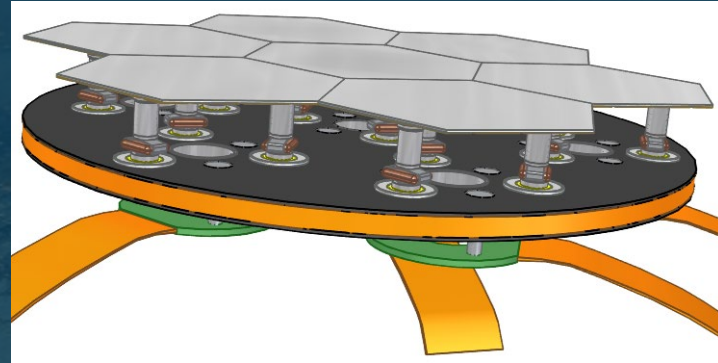
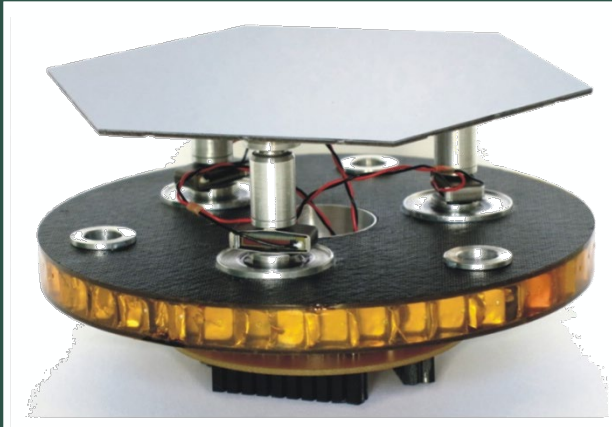
D = 8 m

Deformable Mirrors (I): Principle





Deformable Mirrors (III): examples (Large, small, continuous, segmented, piezo, voice coils etc)



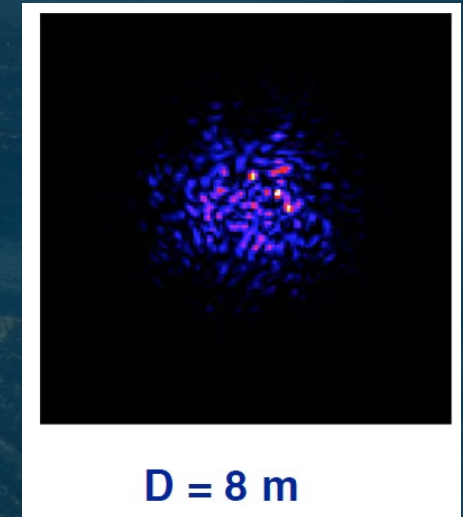
MEMS
1000 actuators

1 cm

Many actuators	Good WFE control	SNR per actuator & Latency & Complexity
Stroke	Large	Slow & Resonance freq. & Required power
Optical coating	Large wavelength band	Complexity
Small DM	Less bulky	More sensitive to aberrations
Performance	Good	Cost

The DM must be properly dimensioned, not always good to have “the best” DM
Rule of thumb: “one actuator per r_0 ”

Number of actuators	depends on the desired correction capability
Stroke	depends on the telescope diameter
Actuator Geometry	should match the WFS sampling geometry
Actuator spacing	related to the stroke
Lowest resonance frequency	should be \gg closed loop rejection bandwidth
Hysteresis	few %
Optical quality	coating, roughness, reflectivity, pupil size, best flat
Thermal stability	should ideally not be sensitive to T°
Control stability	should not drift over time
Electrical properties	e.g power, voltage, current



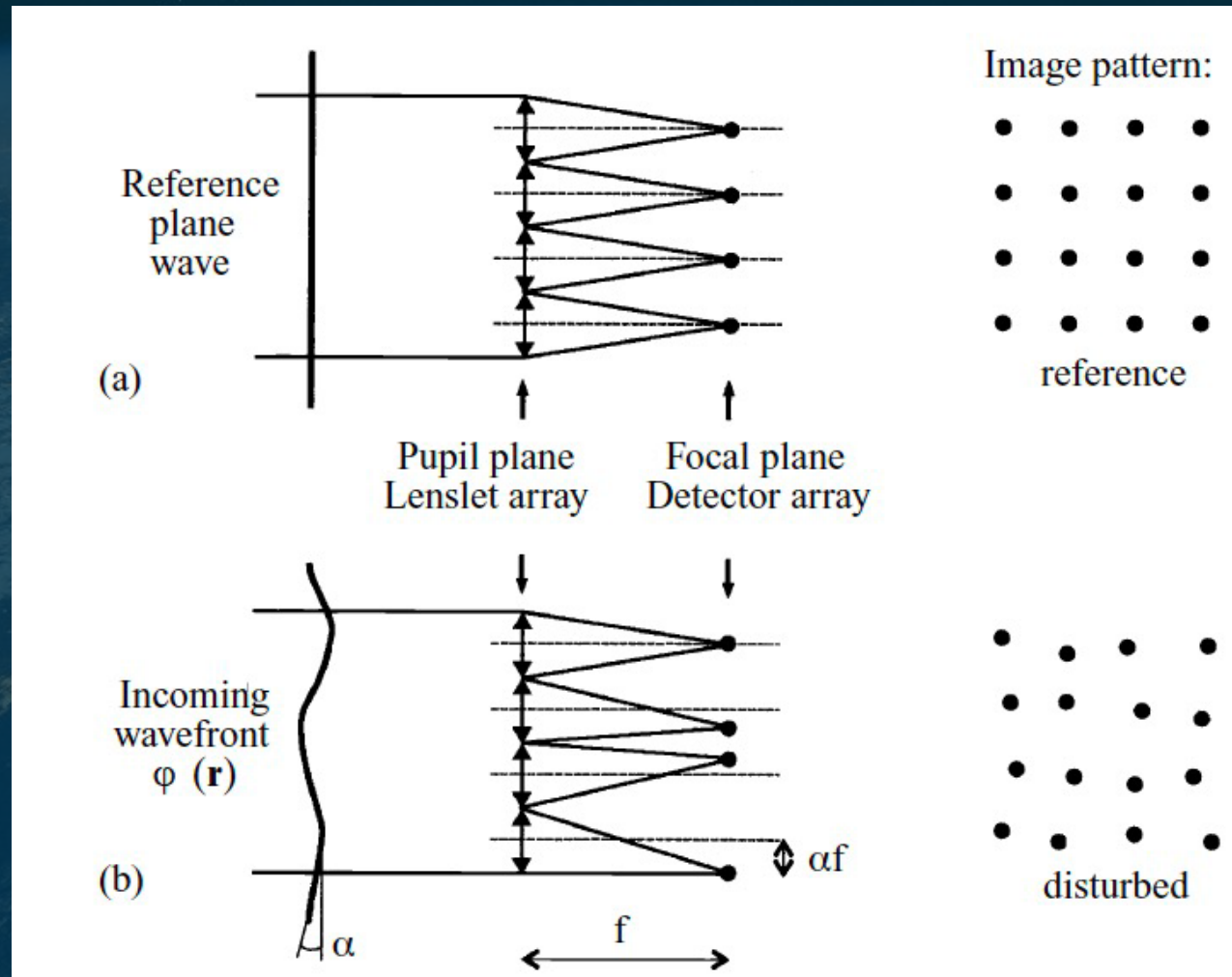
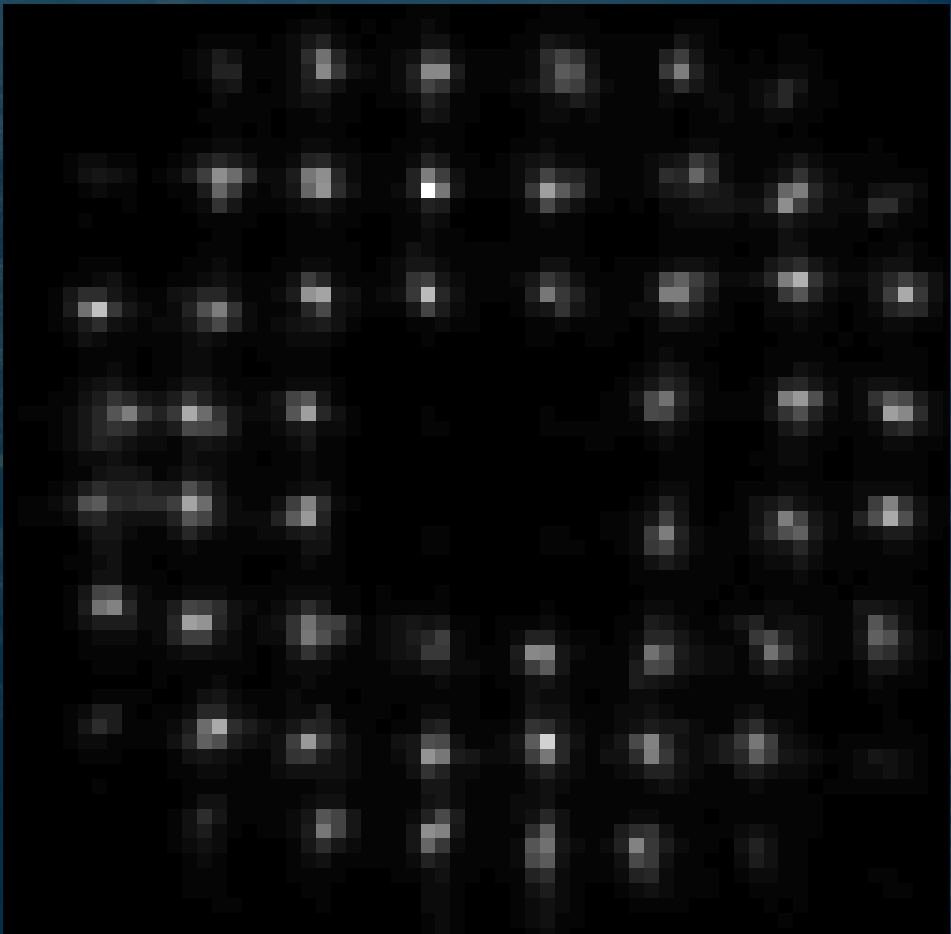
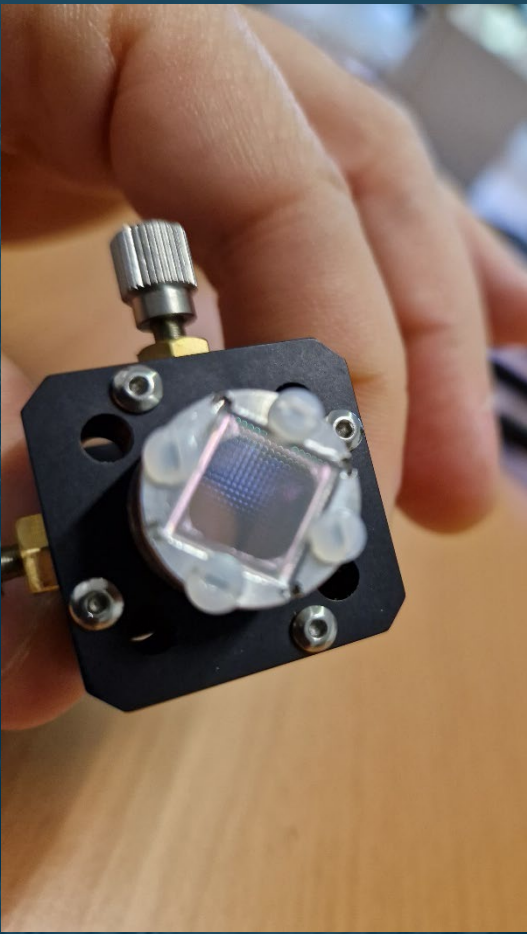


Figure reference: "Adaptive optics in Astronomy" by Francois Roddier

Wavefront sensors (II)



Many microlenslets

Dynamics

Chromaticity

Small WFS

Performance



Good WFE measurement

Large

Large wavelength band

Less bulky

Good



Low SNR per microlens & Latency & Complexity

Slow

Complexity

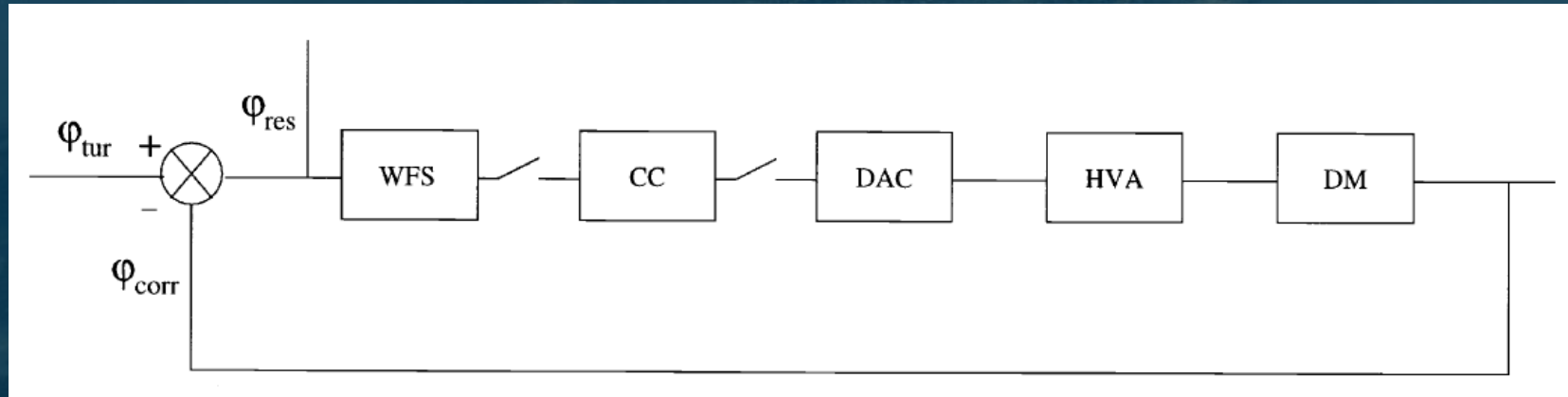
More sensitive to aberrations

Cost

The WFS must be properly dimensioned
Trade-off between spatial & temporal resolution
Rule of thumb: “one lens per actuator on the DM”

Spatial resolution	number of lenses should ideally match that of the DM
Linearity	should be linear function of the input
Dynamic Range	should be able to measure large WFE
Sensitivity	should make efficient use of photons
Speed	should be fast, but fast means lower SNR
Spectral range	should work over a wide enough wavelength range
Latency	should be small
Source	ability to work with extended sources (e.g LGS)

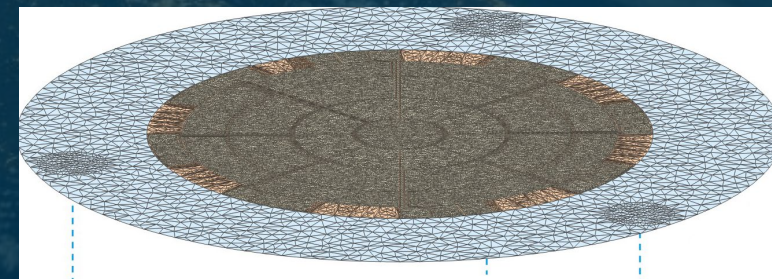
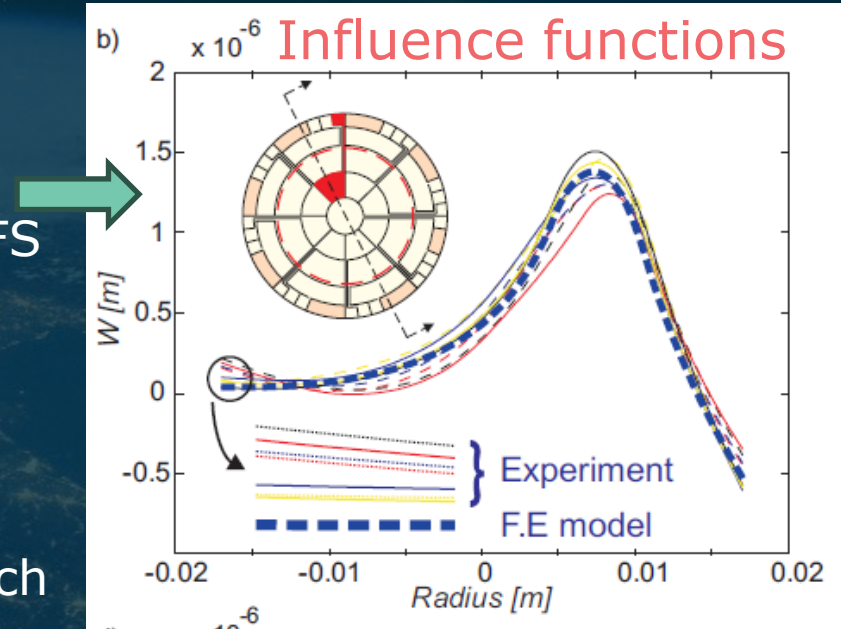
$$\varphi_{\text{res}}(x, y, t) = \varphi_{\text{tur}}(x, y, t) - \varphi_{\text{corr}}(x, y, t) \rightarrow \varphi_{\text{res}} / \varphi_{\text{turb}} \text{ needs to be minimized}$$



Goal: compute the voltages to be applied to the actuators of the DM, in order to deform it to obtain specific target surface.
Mainly used: **zonal** or **modal** control

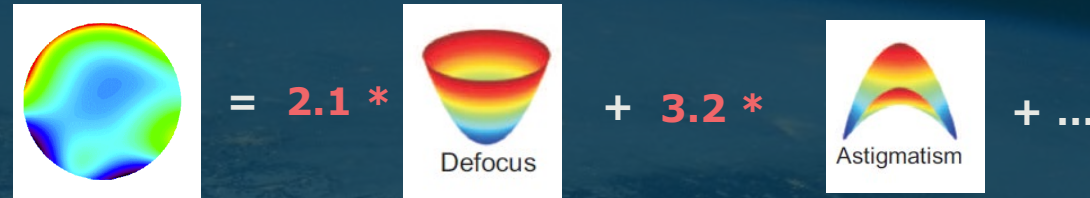
Steps:

1. Influence function: poking each actuator one by one separately with a unit voltage & measuring the local deformation with a WFS
2. Construction of a matrix (\mathbf{J}): one column per actuator and the rows contain the **unitary deformation** in each grid point
3. “We know that when we apply unit voltage, we obtain \mathbf{J} . So which voltage \mathbf{v} should be applied to obtain any other shape \mathbf{w} ?”
 \mathbf{w} contains the local deformations
4. Superposition principle: $\mathbf{w} = \mathbf{J} \mathbf{v}$
 $\rightarrow \mathbf{v} = \mathbf{J}^{-1} \mathbf{w}$ (but \mathbf{J} difficult to invert)



Steps:

0. Any shape can be decomposed in ZM



1. Poking each actuator one by one with a unit voltage & the deformation produced is decomposed in ZM, e.g $2.1 * \text{Defocus} + 3.2 * \text{Astigmatism} + \dots$

2. Construction of a matrix (**J**): one column per actuator and every row contains the **coefficients of each modes (2.1, 3.2, ...)**.

3. “We know that when we apply unit voltage, we obtain this combination of ZM.
So which voltage **v** should be applied to obtain any other combination of ZM, **w** ?”
w contains the ZM coefficients of the desired shape.

4. Superposition principle: $\mathbf{w} = \mathbf{J} \mathbf{v}$
 $\rightarrow \mathbf{v} = \mathbf{J}^{-1} \mathbf{w}$ (but J difficult to invert)

- Correction efficiency given by transfer function of $\Phi_{\text{res}} / \Phi_{\text{turb.}}$
- Major limitation of AO performance:
 - Time delays: $\sigma \propto \frac{\tau_D}{\tau_0}^{5/3}$
 - Control bandwidth frequency: $\sigma \propto \frac{\tau_0}{f_c}^{5/3}$
- Trade-off between:
 - Correction performance & stability
 - Control bandwidth & noise
- Optimization of the loop in real time as a function of the strength of the turbulence
→ Monitoring equipment

- Goal: use bright star in the sky close to the object of interest, to be used as a reference to measure the turbulence with the WFS
- Issue: The amount of bright stars close to the object is small
- Possible solutions:
 - Use the **downlink beam** of a satellite to be used as reference
 - Create an **artificial star** wherever we want !
 - Sodium Guide Star
 - Rayleigh Guide Star
 - **Not use any reference** & increase beam divergence

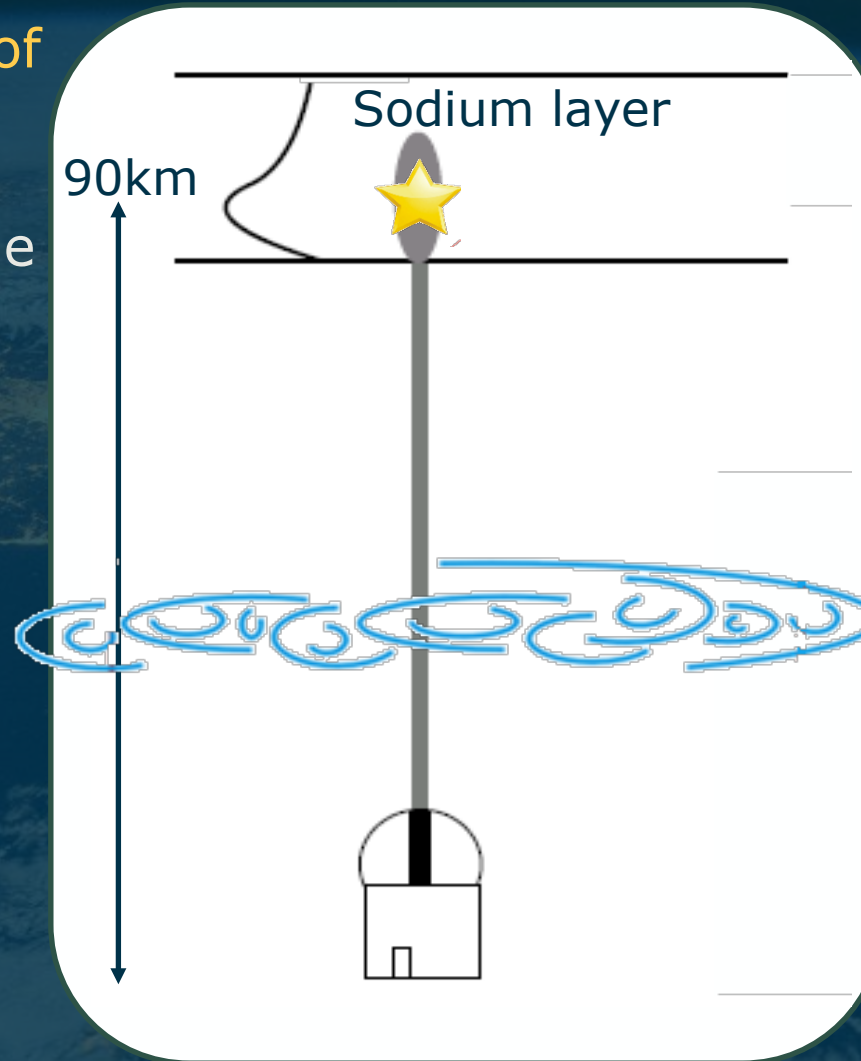


"Artificial Star" (as a reference) – Sodium Guide Star

Excitation of the Na atoms by means of a Specific laser

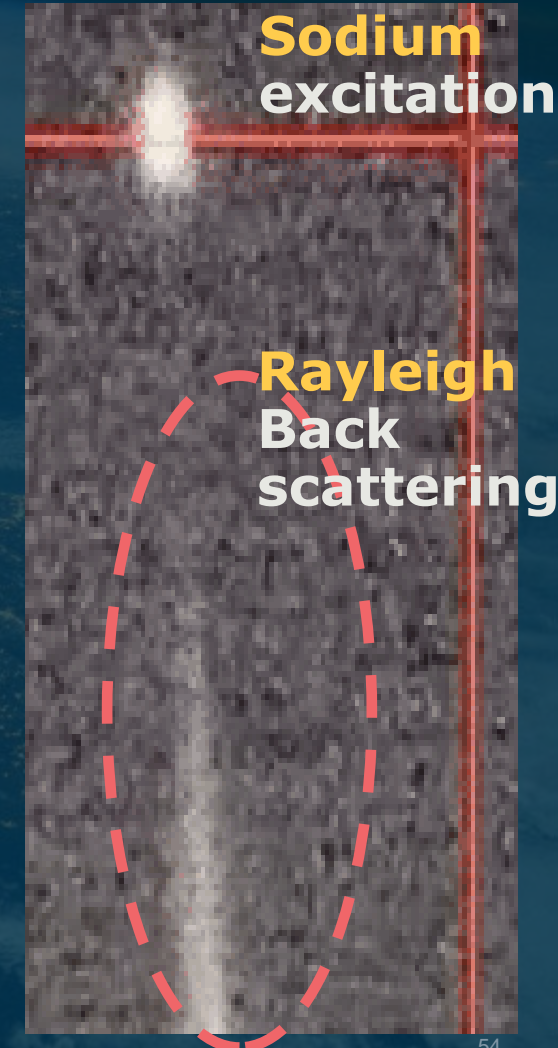
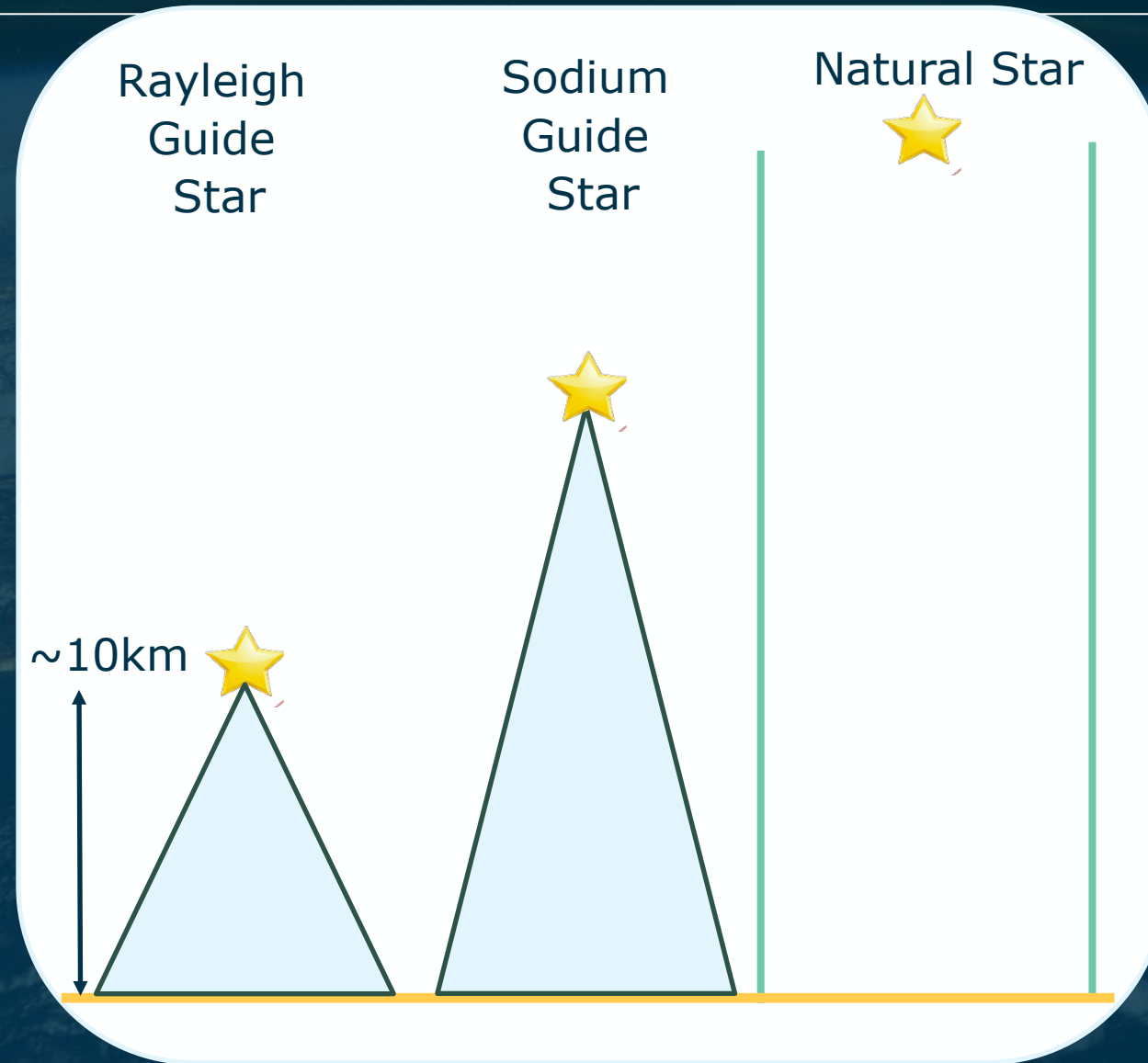
~1980: W. Happer proposes to use the Sodium at 90km (in the Mesosphere)

First used by the US Army Air force then declassified

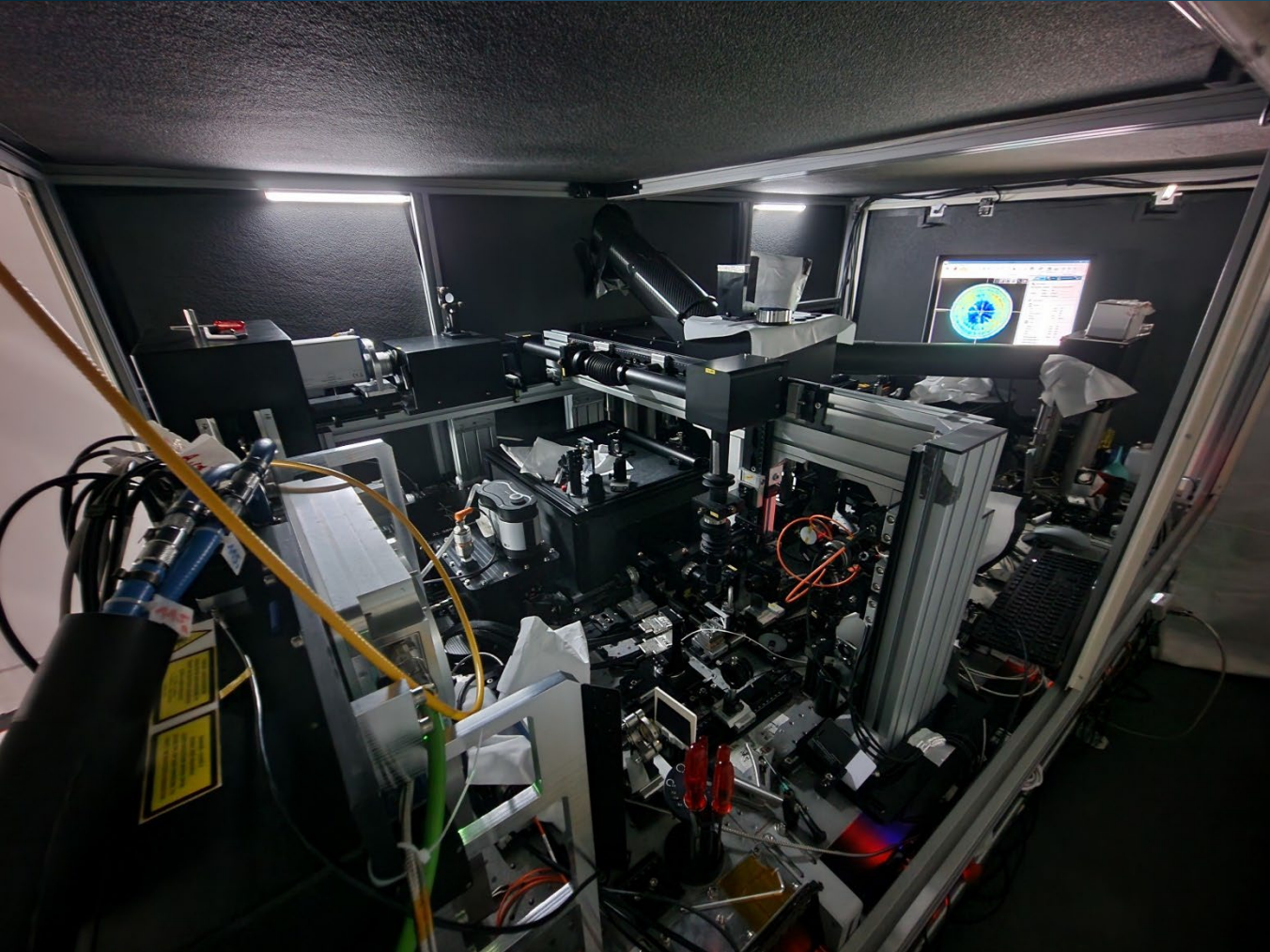


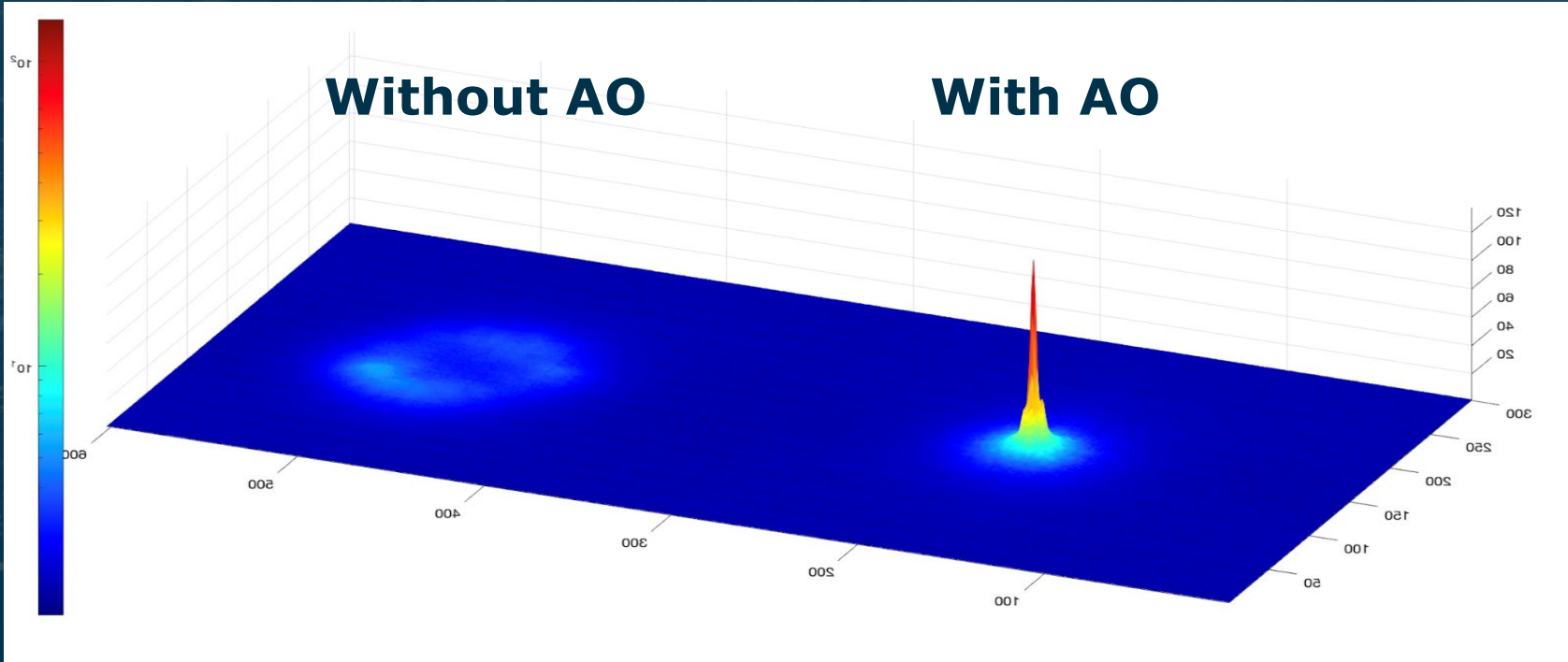
Alternative to Sodium Guide Star: Rayleigh Guide Star

- Any laser
- Using atmospheric back-scattering
- Easier to produce
- But less accurate because it only samples part of the turbulence



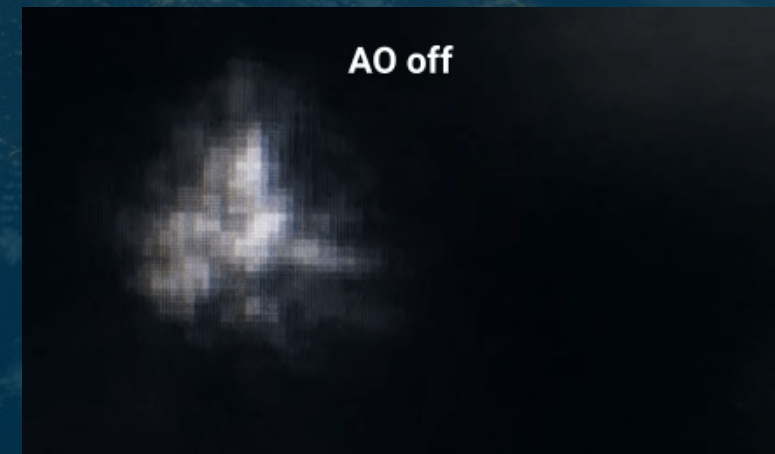
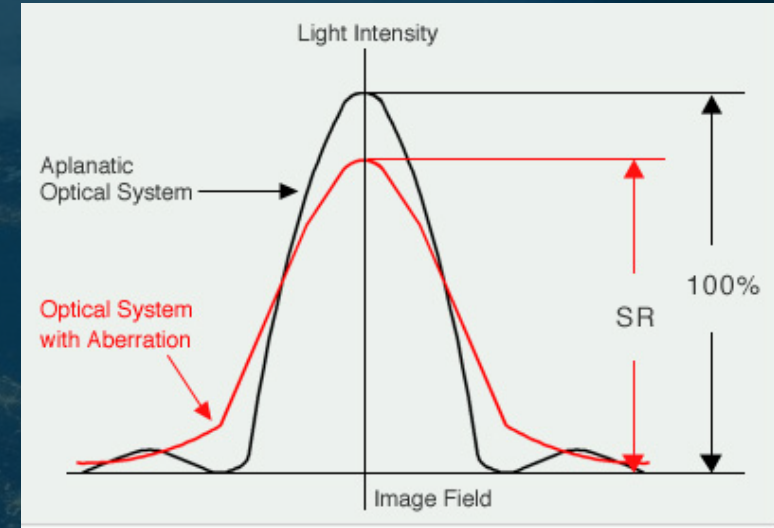
Adaptive Optics: Example (I)





- Performance metrics:
 - **Strehl ratio (SR):** $\frac{\text{Peak intensity actual spot}}{\text{Peak intensity perfect spot}}$
 - **Power coupled in a single mode optical fibre**
(e.g 1m telescope, GEO, 5W downlink, 100s nW, 20% efficiency)
- When AO performs well: more energy in the fibre
- $0 \leq SR \leq 1$ & High SR \rightarrow Better quality
- SR gives an approx. of the coupled power
- Marechal Approx.:

$$SR \approx e^{-\sigma_\varphi^2} \quad \sigma_\varphi = 2\pi \frac{\text{WFE}}{\lambda}$$



Possibilities of traineeships

PhD

PostDoc

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